

How Lotteries Outperform Auctions for Charity*

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Abstract

In their recent paper [Goeree et al. \(2005\)](#) determine that all-pay auctions are better for fundraising activities than lotteries. We show that the introduction of asymmetry among valuations with complete information could reverse this result. Complete information seems well suited to some charity environments.

Keywords: All-pay auctions, charity, complete information, lotteries

JEL Classification: D44, D62, D64

1 Introduction

In their recent paper [Goeree et al. \(2005\)](#) analyze an independent private values model with financial externalities independent of the winner's identity and show that the first-price all-pay auction (hereafter all-pay auction) outperforms lotteries and winner-pay auctions.¹ All-pay auctions combine two effects. On the one hand, like winner-pay auctions, all-pay auctions are efficient. Yet, like lotteries, they are associated with positive externalities – or a return – from the losers' bids. However, lotteries seem to be used more frequently by fundraisers than auctions. Maybe some element, as asymmetry or heterogeneity among participants, missing in [Goeree et al.](#)'s analysis could explain this common use of lotteries.

In this note we focus on all-pay auctions and lotteries as their revenue is not bounded – in contrast to winner-pay auctions.² Is it still true that all-pay auctions are better at

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¹They also show that the lowest-price all-pay auction with the suitable fees and reserve price is an optimal fundraising mechanism. [Engers and McManus \(2007\)](#) determine other optimal auctions for charity.

²See [Bos \(2008\)](#) for a comparison between all-pay auctions and winner-pay auctions with asymmetric bidders under complete information.

raising money for charity with asymmetric participants? Agents can be asymmetric in different ways. By instance, their valuations can be drawn from different distributions with incomplete information or their valuations can be strictly ordered with complete information. Both complete information and incomplete information are well suited to different kinds of fundraising environments. For example, complete information could occur in some charity dinners among people of the same social class who know each other well and also in some voluntary social organizations. On the other hand, incomplete information is found in some fundraising activities on the Internet.³ As few results are available on asymmetric all-pay auctions in an independent private values model, we investigate complete information. We show that the introduction of asymmetry among valuations with complete information can reverse this result of [Goeree et al. \(2005\)](#).

This paper is organized as follows. In the next section we give a presentation of the framework and the raffle. In section 3 we compare lottery and all-pay auction revenues. We do this using [Bos' \(2008\)](#) results on all-pay auctions for charity with complete information.

2 The Lottery

We consider that a fundraiser has one item she is going to sell in a local community of n people by means of a lottery. In this community, people know each other well so the valuation of participant i , denoted v_i , and the ranking of the valuations $v_1 > v_2 > v_3 \geq \dots \geq v_n$ are common knowledge.⁴ As the item is sold to collect money for a charity purpose, participants get an additional benefit from the revenue raised. Let α denote the altruism parameter or the return to participants of the fundraising activity. As in [Goeree et al. \(2005\)](#), participants are not completely altruistic, which means that their altruism parameter is strictly less than one - and positive.⁵

To the best of our knowledge, [Morgan \(2000\)](#) was the first to study lotteries as a fundraising mechanism. Yet, unlike [Morgan \(2000\)](#) here the asymmetry is on the valuations and not the altruism parameters. We denote x_i the number of tickets bought by player i such that the revenue collected is $R^{LOT} \equiv \sum_{i=1}^{n^p} x_i$ with $n^p \leq n$ being the number of active participants. Thus, the expected utility of i is

$$EU_i(x_i, x_{-i}) = v_i \frac{x_i}{\sum_{j=1}^n x_j} - x_i + \alpha \sum_{j=1}^n x_j$$

where $\frac{x_i}{\sum_{i=1}^n x_i}$ is the probability of winning for potential participant i and $\alpha \sum_{i=1}^n x_i$ the return

³See for example, the eBay charity auction website *Giving Works*, ebaygivingworks.com, and the charity lotteries held on charityfacts.org.

⁴This ranking is relevant with the purpose of this paper which is to determine the consequence of heterogeneity. Moreover, consider at least the three highest valuations are strictly ordered avoid the multiplicity of the equilibria in the all-pay auction.

⁵This assumption is straightforward with the framework of [Bos \(2008\)](#) that we used to compare the revenue with all-pay auctions. Technically, this assumption means that the utility of participants decreases if the amount of money they spend (their bid or the number of tickets bought) increases.

he gets from the amount raised. Following [Morgan \(2000\)](#), the set of first order conditions is given by

$$\frac{\sum_{j=1, j \neq i}^n x_j}{\left(\sum_{j=1}^n x_j\right)^2} v_i - (1 - \alpha) \leq 0 \quad \forall i$$

with equality if i is an active participant which means that $x_i > 0$ and otherwise strictly below zero. This leads to the following result:

Proposition 1. *The lottery has a unique Nash equilibrium such that the number of tickets bought by the participants is given by $x_i = \begin{cases} \frac{n^p-1}{1-\alpha} \frac{1}{\sum_{j=1}^{n^p} 1/v_j} \left(1 - \frac{n^p-1}{v_i \sum_{j=1}^{n^p} 1/v_j}\right) & \forall i \leq n^p \\ 0 & \text{otherwise} \end{cases}$ and*

the revenue raised is $R^{LOT} = \frac{n^p-1}{1-\alpha} \frac{1}{\sum_{i=1}^{n^p} 1/v_i}$ with n^p the highest integer of $m \in \{2, \dots, n\}$ which satisfies $m \leq 2 + v_m \sum_{i=1}^{m-1} \frac{1}{v_i}$.

As our lottery is similar to a Tullock contest ([Tullock, 1980](#)), the equilibrium is unique and there are at least two participants in the lottery: these are the two members of the community with the two highest valuations who take part. Moreover, if a participant with a valuation v_i is active, then all the participants with higher valuations are also active. We omit the proof for these results as the number of tickets bought by the participants and the revenue raised are a slight variation on the proof of [Corchón \(2007\)](#) and the number of participant a similar computation of the proof of [Meland and Straume \(2007\)](#).

3 Revenue Comparison

In the following, we use [Bos' \(2008\)](#) results on all-pay auctions for charity to compare them with our result on lotteries. [Bos](#) determines the unique Nash equilibrium in the all-pay auctions with financial externalities. It is a mixed strategies equilibrium where only the two bidders with the highest valuations participate. Then, the all-pay auction expected revenue is not bounded, as is the revenue of the lottery. The results are summed up in the following table where $\mathbb{E}R^{AP}$ is the expected revenue of the all-pay auction:

Ranking of values	R^{LOT}	$\mathbb{E}R^{AP}$
$v_1 = \dots = v_n \equiv v$	$\frac{n-1}{n} \frac{v}{1-\alpha}$	$\frac{v}{1-\alpha}$
$v_1 > v_2 > \dots \geq v_n$	$\frac{n^p-1}{1-\alpha} \frac{1}{\sum_{i=1}^{n^p} 1/v_i}$	$\frac{1}{2} \frac{v_2}{1-\alpha} \left(\frac{v_2}{v_1} + 1\right)$

Table 1: Revenue and expected revenue for each design

Suppose that participants have the same valuations. Then, it follows that all-pay auctions lead to higher revenues than lotteries for charity. In fact, we find the same qualitative results as [Goeree et al. \(2005\)](#) and confirm the ones of [Orzen \(2005\)](#) who compared all-pay auctions and lotteries with complete information in a different framework. However, as seen in the next

proposition, this result does not hold when the asymmetry between the active participants is strong enough.

Proposition 2. *Lottery is better at raising money for charity than all-pay auction if and only if the participants with the two highest valuations are asymmetric enough.*

Proof. For $n^p = 2$, $R^{LOT} > \mathbb{E}R^{AP}$ is true if and only if $\frac{1}{\frac{1}{v_1} + \frac{1}{v_2}} > \frac{1}{2} \frac{v_2}{v_1} (v_2 + v_1)$

$$\begin{aligned} &\Leftrightarrow 1 > \frac{v_2^2}{v_1^2} + 2 \frac{v_2}{v_1} \\ &\Leftrightarrow v_1 - v_2 > \frac{v_2}{v_1} (v_2 + v_1) \end{aligned} \quad (1)$$

For $n^p = 3$, $R^{LOT} > \mathbb{E}R^{AP}$ is true if and only if $2 \frac{1}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}} > \frac{1}{2} \frac{v_2}{v_1} (v_2 + v_1)$

$$\begin{aligned} &\Leftrightarrow 3 > \frac{v_2^2}{v_1^2} + 2 \frac{v_2}{v_1} + \frac{v_2}{v_1} \frac{v_1 + v_2}{v_3} \\ &\Leftrightarrow 3v_1^2 - v_2^2 - 2v_1v_2 > v_1v_2 \frac{v_1 + v_2}{v_3} \\ &\Leftrightarrow v_1 - v_2 > \frac{v_1v_2}{v_3(3v_1 + v_2)} (v_2 + v_1) \end{aligned}$$

Finally, for $n^p > 3$, $R^{LOT} > \mathbb{E}R^{AP}$ is true if and only if $\frac{n-1}{\sum_{i=1}^{n^p} \frac{1}{v_i}} > \frac{1}{2} \frac{v_2}{v_1} (v_2 + v_1)$

$$\begin{aligned} &\Leftrightarrow 2(n^p - 1)v_1 > \frac{v_2^2}{v_1} + 2v_2 + v_1 + v_2(v_2 + v_1) \sum_{i>2}^{n^p} \frac{1}{v_i} \\ &\Leftrightarrow 2(v_1 - v_2) > 2(-n^p + 3) \frac{v_2^2}{v_1} - (2n^p - 5) \frac{v_1^2 - v_2^2}{v_1} (v_2 + v_1) \sum_{i>2}^{n^p} \frac{1}{v_i} \\ &\Leftrightarrow (v_1 - v_2) \frac{(2n^p - 3)v_1 + (2n^p - 5)v_2}{v_1} > 2(-n^p + 3) \frac{v_2^2}{v_1} + v_2(v_2 + v_1) \sum_{i>2}^{n^p} \frac{1}{v_i} \quad (2) \\ &\Leftrightarrow v_1 - v_2 > \frac{1}{\prod_{i>2}^{n^p} v_i} \frac{v_2}{(2n^p - 3)v_1 + (2n^p - 5)v_2} \left(-2(n^p - 3)v_2 \prod_{i>2}^{n^p} v_i + v_1(v_2 + v_1) \sum_{k>2} \prod_{i>2, i \neq k}^{n^p} v_i \right) \end{aligned}$$

■

If our framework is well suited to certain charity settings (*e.g.* dinners held by a local Rotary Club), the introduction of asymmetry among participants contradicts Goeree et al.'s (2005) qualitative results. Here, we assume that the asymmetry arises from the valuations. Actually it could also be due to the altruism parameters. Yet, it is just another way of presenting the problem and leads to similar qualitative results.

The next corollary analyzes the effect of the asymmetry among the participants with the second highest valuation and the ones with lower valuations on the relative revenues obtained

with the all-pay auction and the lottery. In the following we assume that $v_i = \lambda v_2$ for $i > 2$ and $\lambda \in [0, 1)$. The lower λ the higher the asymmetry (or the distance) among the participants with the second highest valuation and the ones with lower valuations. We call this distance the secondary level of asymmetry.

Corollary 1. *Let us assume that $v_1 > v_2 > v_i = \lambda v_2$ for $i > 2$ and $\lambda \in [0, 1)$. The secondary level of asymmetry sets that there are either 2 or n participants. In the former case the asymmetry between the two highest valuations such that lottery outperforms all-pay auction is independant of the secondary level of asymmetry. In the latter case, the higher the secondary level of asymmetry the higher the asymmetry between the two highest valuations needs to be for lottery outperforms all-pay auction.*

Proof. As $v_i = \lambda v_2$ for $i > 2$, n^p is the highest integer of $m \in \{2, \dots, n\}$ such that $m \leq 2 + v_m \sum_{i=1}^{m-1} \frac{1}{v_i}$. For all $m \in \{3, \dots, n\}$ it follows $m \leq 2 + \lambda v_2 \left(\frac{1}{v_1} + \frac{1}{v_2} \right) + m - 3$. Then, $n^p = n$ if $\lambda \in \left[\frac{v_1}{v_1 + v_2}, 1 \right)$. Otherwise, $n^p = 2$ as $2 \leq 2 + \lambda v_2 \left(\frac{1}{v_1} + \frac{1}{v_2} \right)$.

The secondary level of asymmetry sets the number of participants. Thus, the number of participants determines the level of asymmetry between the two highest values such that lottery outperforms all-pay auction. Clearly, for $n^p = 2$ as we can see in (1) the threshold is constant. For $n^p = n$, (2) leads to

$$(v_1 - v_2) \frac{(2n - 3)v_1 + (2n - 5)v_2}{v_1} > -2(n - 3) \frac{v_2^2}{v_1} + (n - 2) \frac{v_1 + v_2}{\lambda}.$$

Hence,

$$v_1 - v_2 > \frac{-2(n - 3)v_2^2 + \frac{(v_1 + v_2)v_1(n - 2)}{\lambda}}{(2n - 3)v_1 + (2n - 5)v_2}$$

For $n^p = n$, the smaller the secondary level of asymmetry (which means the bigger λ), the smaller the distance between the two highest values needs to be for lottery outperforms all-pay auction. ■

The number of participants is 2 if λ is lower than $\frac{v_1}{v_1 + v_2}$ and n otherwise. Then, the number of participants determines the level of asymmetry between the two highest values such that the lottery outperforms the all-pay. These levels of asymmetry are depicted in Figure 1.

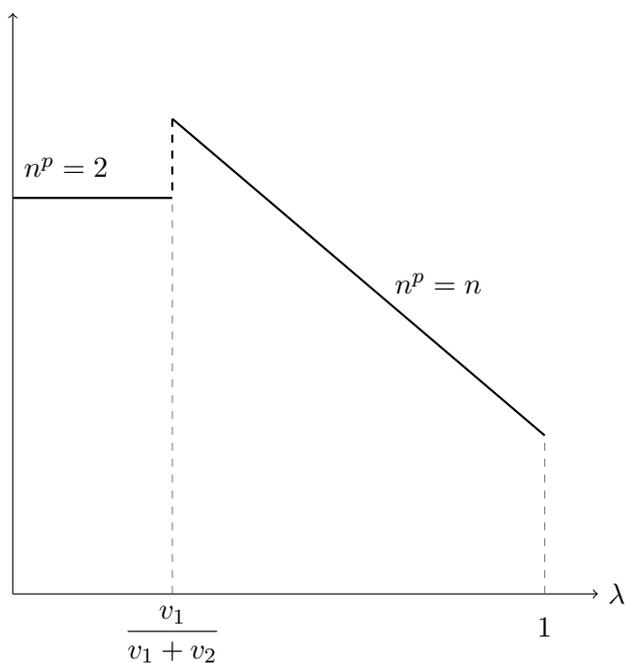


Figure 1: Level of asymmetry

4 Conclusion

In this paper we show that lotteries could be better at raising money for charity than all-pay auctions when participants are asymmetric enough. Moreover, as lottery revenues are not bounded, this mechanism seems more appropriate than auctions for fundraising activities.

This work could be rounded out by a laboratory experiment. Only two lab experiments have been led to compare lotteries and all-pay auctions. [Onderstal and Schram \(2009\)](#) compared lotteries, first-price all-pay and winner-pay auctions within the framework of [Goeree et al. \(2005\)](#) while [Orzen \(2005\)](#) runs an experiment with a complete information framework for symmetric participants. The former confirms the theory while the latter is inconclusive. As asymmetry can change the theoretical results it would be interesting to conduct new experiments with asymmetric participants.

Finally, this paper leaves open for future research the question of fundraising mechanisms with asymmetric participants under incomplete information.

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