# Contestants with Inherent Efforts*

Olivier Bos†

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**Abstract**

We study an *ex ante* correlation between minimum efforts and the valuation of contestants in an all-pay auction with two heterogeneous participants. The unique Nash equilibrium is characterized and the expected rent determined. This analysis is relevant for contests in which some participants benefit from a behavioral tendency such as TV singing or lobbying contests.
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## 1 Introduction

In many contest settings, participants do not have the same *ex ante* probability of winning. Not only might they evaluate the prize for winning differently but some contestants may also benefit from intrinsic incentives. This means that they will always produce an effort greater than a minimum threshold because of moral and mental reasons. This kind of phenomenon can be observed for example in TV singing contests. Some participants would feel shameful if they will not reach a certain level. Consequently, they cannot sing at a lower level of performance than their natural tendency makes it possible. It seems behaviorally relevant to consider that an *ex ante* higher minimal effort due to morality leads to a higher evaluation of the contest winning prize independently of the other contestants' valuation. Indeed, participants believe they have to perform starting a certain level at some activities might evaluate these activities even more.[1] Another situation to observe this kind of phenomenon are lobbying process. Lobbyists know *ex ante* they would never propose less than a certain payment. It seems also behaviorally consistent to consider a higher minimal predisposition payment leads to a higher evaluation of the contest winning prize awarded.

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†Panthéon-Assas University. Address: Panthéon-Assas University – LEMMA, 4 rue Blaise Desgoffe, 75006 Paris France. E-mail: olivier.bos@u-paris2.fr. Tel: +33170232005.

[1] There is a conceptual difference between a minimum effort and a head start. With a minimum effort, contestants cannot provide a lower effort than a certain threshold due to inherent incentives, yet they bear its entire cost. With a head start each contestant has only to support the cost of efforts going beyond a certain threshold. That might be due to training, practice or related to an inherent talent. Moreover, head starts do not involve any correlation between a minimum effort and the contestant's valuation. Another example to understand the differences between minimum efforts and head starts in contests are running races. In our case, no participant would ever run below a certain speed than their moral tendency makes it possible. Yet, they have to support their own effort to be able to run in this way. Thus, higher minimal speed leads to a higher winning prize evaluation. With head starts, runners do not support the effort going beyond a certain speed. That is the consequence of their training and does not affect their own evaluation of the winning prize. Head starts are well analyzed in all-pay auctions and more generally in contests, e.g. Konrad (2002), Li and Yu (2012), Siegel (2014) and Denter and Sisak (2016).

There is no paper analyzing an *ex ante* correlation among minimum positive efforts and the own valuation of contestants, which seems relevant in some cases. This paper proposes to investigate this question in a simple all-pay auction model with two heterogeneous contestants. The unique Nash equilibrium and the expected rent at the equilibrium are determined.

## 2 The Model

Two risk-neutral contestants are competing for a prize. This prize might be an award in a singing tournament on TV or a government allocation using a lobbying system. Therefore, contestants could be either fair competitors in a contest or lobbyists. The organizer of the contest awards the prize to the participants who makes the greatest effort.

We suggest analyzing this contest through an all-pay auction. Contestants 1 and 2 have different valuations for the prize $v_1 \geq v_2 > 0$ which are assumed to be common knowledge. Contests such as all-pay auctions with no inherent efforts have been widely analyzed. The foremost papers on this topic are Hillman and Riley (1989) and Baye et al. (1996). If contestant $i$ wins, she gets a payoff $v_i - x_i$ with $x_i$ as her effort. Otherwise, she loses her effort expended $x_i$ without obtaining any positive reward. In our setting, the more highly the contestant's value for the prize the higher is her natural gift to provide a starting effort. Therefore, contestant $i$ would make her initial effort greater than or equal to $tv_i$ such that $t \in [0, 1)$. Notice that $t \geq 1$ is not appropriate here. If it were, the minimum effort could be higher than the maximum effort. Moreover if $t = 0$, the usual results from all-pay auctions follow.

Consequently there is no pure strategy Nash equilibrium for similar reasons to those found in an all-pay auction with no minimum positive efforts. We look then for Nash equilibria in mixed strategies. We denote $F_i(x) \equiv \mathbb{P}(X_i \leq x)$ the cumulative distribution functions such that contestant $i$ decides to make an effort less than $x$.

## 3 Results

Once the unique Nash equilibrium in mixed strategies and the expected rent at the equilibrium have been determined, we can understand how minimum efforts affect the contest. This could have important policy implications for the contest designer. To establish the equilibrium we need to proceed in two preliminary steps: first what is the support of every positive density; Second are there any mass points on the support and if so which ones?

**Lemma 1.** *At equilibrium, the minimum efforts are $tv_1$ for contestant 1 and $tv_2$ for contestant 2. Contestant 2 has a density equal to zero on the support $(tv_2, tv_1]$.*

With probability one, contestant $i$'s offer will be at least equal to $tv_i$, which means that $\min x_i \geq tv_i$. In the following, let us assume that $\min x_1 = x > tv_1$. Then $\mathbb{P}(X_1 < \{x\}) = 0$. Indeed contestant 1 never makes any offer in the interval $[tv_1, x)$. Her competitor offers either $tv_2$ or $x + \varepsilon$ for $\varepsilon > 0$, an effort between these two values being strictly dominated. Consequently, contestant 1 can make an effort $x - \varepsilon$ without altering her probability of winning. Therefore, her minimum effort is $tv_1$.

Moreover, making an effort in the interval $(tv_2, tv_1]$ is strictly dominated for contestant 2. Hence, $\mathbb{P}(tv_2 < X_2 \leq tv_1) = 0$ and she can only lose for all effort $tv_2 < x \leq tv_1$. Indeed, when she offers $x = tv_2$, she does not affect her probability of winning but increases her payoff. Furthermore, she can increase her probability of winning by offering $x = tv_1 + \varepsilon$ for $\varepsilon > 0$. Therefore, contestant 2's density function is zero on the interval $(tv_2, tv_1]$.

**Lemma 2.** *At equilibrium, contestants offer the same maximum effort $v_2$. Every contestant has a mass point for his minimum effort and a mass point can never be on $(tv_1, v_2]$.*

Given the former analysis in Lemma 1, contestant 2 has a mass point on $tv_2$. Contestant 2's strategy space is $\{tv_2\} \cup (tv_1; v_2]$. For reasons similar to those in Hillman and Riley (1989), having a mass point on the contestants' common strategy set is dominated for every contestant (since they deviate).[2]

Let us now only consider the contestants' common strategy set, that is to say $(tv_1; v_2]$. A contestant's equilibrium payoff is a constant function of her entire strategy set.[3] Hence,

$$F_2(x)v_1 - x = v_1 - v_2, \tag{1}$$

for all $x \in (tv_1; v_2]$. The left hand side of this equation is contestant 1's expected utility for all efforts in $(tv_1; v_2]$, the right hand side is contestant 1's payoff when she provides an effort $v_2$. In the same way, contestant 2's effort is such that

$$F_1(x)v_2 - x = 0, \tag{2}$$

for all efforts in $\{tv_2\} \cup (tv_1; v_2]$.

In particular, for all efforts in the interval $(tv_1, v_2]$ it follows that

$$v_2(1 - F_1(x)) = v_1(1 - F_2(x)).$$

As contestant 2 has a mass point on $tv_2$, the limit in $tv_1$ yields the following result[4]

$$F_1(tv_1) = 1 - \frac{v_1}{v_2} + \frac{v_1}{v_2}F_2(tv_1).$$

Using (1) and (2) it is easy to determine the contestants' distribution functions, available in Proposition 1 hereafter. Interestingly, as $F_2(tv_1)$ is not equal to $1 - \frac{v_2}{v_1}$, contestant 1 has a mass point on $tv_1$. The agents' distribution functions are drawn in Figure 1.

**Proposition 1.** *There is a unique Nash equilibrium. The contestants' strategies for all $x \in (tv_1; v_2]$ are*

$$F_1(x) = \frac{x}{v_2} \text{ and } F_2(x) = 1 + \frac{x - v_2}{v_1}.$$

*Each contestant has one mass point: that is in $tv_1$ for contestant 1 and $tv_2$ for contestant 2.*

The contestants' decision to participate is provided by the probabilities

$$1 - F_1(tv_1) = 1 - \frac{tv_1}{v_2} \text{ and } 1 - F_2(tv_2) = \frac{v_2 - tv_1}{v_1}$$

The expected rent at the equilibrium can now be determined. Obviously, if the maximum effort $v_2$ is below contestant 1's minimum effort $tv_1$, offering more than their minimum effort is dominated for all contestants. Hence, $\mathbb{E}R = t(v_1 + v_2)$ for all $t \geq \bar{t}$ where $\bar{t} \equiv \dfrac{v_2}{v_1}$.

[2] We provide here a well-known argument (see for instance Che and Gale (1998)) to support this idea. If only one contestant has a mass point on the support that is common to both contestants, her competitor's density function below this mass point is equal to zero. Hence, she is going to move and her mass point will be the support's lower bound. This action does not affect her probability of winning, but it increases her payoff if she wins. In a similar way, if contestants have a mass point, deviating increases their probability of winning. Consequently, the result follows.

[3] In the following we use similar technical arguments to Che and Gale (1998).

[4] As $\mathbb{P}(tv_2 < X_2 \leq tv_1) = 0$ it follows that $\lim_{x \to tv_1} F_2(x) = F_2(tv_1) = F_2(tv_2)$.

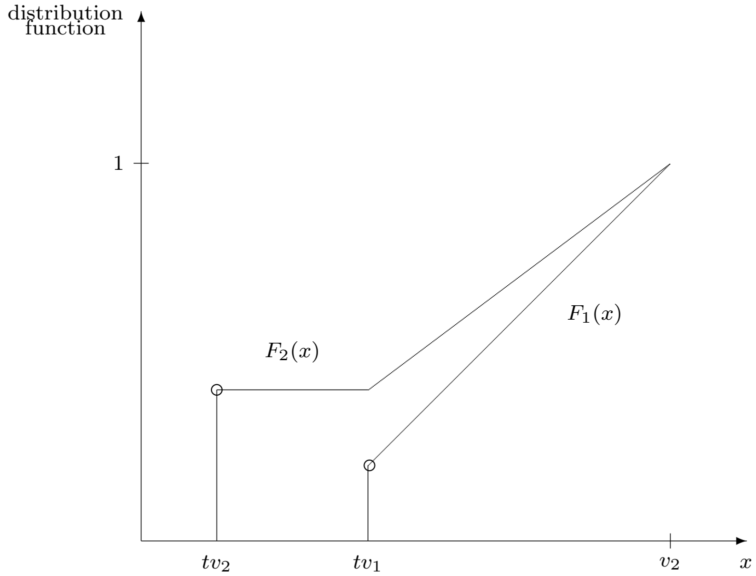


Figure 1: Cumulative distribution functions at the equilibrium

**Proposition 2.** *Given the distribution functions $F_1(.), F_2(.)$ at equilibrium, the expected rent at the equilibrium is*

$$\mathbb{E}R = \begin{cases} v_2 \dfrac{v_1 + v_2}{2v_1} + (tv_1)^2 \dfrac{v_1 - v_2}{2v_1 v_2} + tv_2 \left(1 + \dfrac{tv_1 - v_2}{v_1}\right) & \textit{if } t < \bar{t} \\ t(v_1 + v_2) & \textit{otherwise} \end{cases}$$

*Proof.* We only have to compute the individual expected equilibrium expenditures when $t < \bar{t}$:

$$\begin{aligned} \mathbb{E}R_i &= \int_{tv_1}^{v_2} x f_i(x) dx + tv_i F_i(tv_1) \\ &= v_2 \int_{tv_1}^{v_2} f_i(y) dy - \int_{tv_1}^{v_2} \int_{tv_1}^{x} f_i(y) dy dx + tv_i F_i(tv_1) \\ &= v_2 (F_i(v_2) - F_i(tv_1)) - \int_{tv_1}^{v_2} F_i(x) - F_i(tv_1) dx + tv_i F_i(tv_1) \\ &= v_2 - \int_{tv_1}^{v_2} F_i(x) dx + (tv_i - tv_1) F_i(tv_1) \end{aligned}$$

Hence, $\mathbb{E}R_1 = \dfrac{v_2^2 + (tv_1)^2}{2v_2}$ and $\mathbb{E}R_2 = \dfrac{v_2^2 - (tv_1)^2}{2v_1} + tv_2 \left(1 + \dfrac{tv_1 - v_2}{v_1}\right)$ ■

If $t \leq \bar{t}$, the expected rent is increasing with the inherent effort parameter $t$ as the lower bound of the contestant's support increases. Consequently, the correlation between contestants' valuations and minimum efforts is an important policy variable for the contest designer.

# 4 Conclusion

We have shown that minimum efforts affect the contestants' strategies at the equilibrium and can *ex ante* improve the expected rent. This could have consequences for the selection of potential participants in a contest. The contest designer should not only look at the contestants valuations but also at another characteristic, the correlation between the contestants' valuations and minimum efforts.

We expect that these results could be used for future research. The contest designer could learn sequentially the minimum effort and the valuation of each potential participants. Then, an interesting investigation would be to determine how releasing information about minimum efforts could affect the selection of contestants. Another follow-up inquiry concerns incomplete information on minimum efforts and valuations. A relevant way to keep the interdependency between these two private parameters would be to assume they are positive *affiliated*.[5]

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[5]Affiliation is a strong form of positive correlation. See Milgrom and Weber (1982) for a discussion about affiliation.