Agreeing on efficient emissions reduction: *Online appendix**

Olivier Bos† Béatrice Roussillon‡ Paul Schweinzer§

This online appendix contains proofs, further details, and examples which could—due to space restrictions—not be included in the main manuscript. We identify equation (n) in the main text using the reference M:(n); all other references are made to this appendix.

Appendix A: Extensions and robustness

A.1 Enforcing participation in the deterministic output model

The purpose of this section is to show that a simple way to deter free-riding of individual nations even in the deterministic output version of our model is to grant most favoured 'green' trading terms only to participating nations. Similarly, environmental certification conditional on treaty commitment can be a powerful complementary tool to enforce participation. We would like to point out prominently, however, that participation in the deterministic model version is generally not renegotiation proof. This is the reason why we introduce the more realistic, stochastic output version of our model in section 4 of the main text which supports agreement formation through an insurance motive.

The expansion of equilibrium abatement efforts from their no agreement level f^0 to the level within the agreement f^* creates 'green' products; the higher abatement efforts, the greener is productive output. The idea is to label free-riding countries' products and thereby creating (political) incentives to respect commitments to the IEA. Consequently, firms' lobbying against environmental regulation may result in that country's desertion followed by the IEA labelling its goods. We thus propose a negative label which signals a product lacking the 'green' environmental standards enforced by the IEA.[1]

This idea can be formalised in our setup by decreasing the value of a deserting country's output. Loosely speaking, we require that—once certified as environmentally unfriendly—a consumer's

*†LEMMA-Department of Economics, Panthéon-Assas University, 4 rue Blaise Desgoffe, 75006 Paris, France, olivier.bos@u-paris2.fr. ‡Univ. Grenoble Alpes, GAEL, F-38000 Grenoble, France and INRA, GAEL, F-38000 Grenoble, France beatrice.roussillon@upmf-grenoble.fr. §Department of Economics, University of York, Heslington, York YO10 5DD, United Kingdom, paul.schweinzer@york.ac.uk. (19-Jul-2015)

[1] There are many green labelling examples: UK supermarket chain Tesco has recently introduced a promotional campaign on carbon labels. US Walmart and French Casino have similar ambitions. Examples of negative labelling campaigns are the mandatory GMO labelling implemented in Europe, and "we're Greenpeace, and we want a fresh green Apple" targeted at US computer maker Apple. Grankvist et al. (2004) argue that negative labelling may have a higher consumption impact than positive labelling. Engel (2004) underlines the necessity to inform consumers, especially when a firm is found cheating on its environmental claims.

willingness to pay for labelled products decreases.[2] Therefore, the revenue generated from the production of labelled products sinks as these products suffer a decrease in price of $x(\boldsymbol{f}) \in [0,1]$. This fraction corresponds to the deserter's deviation from agreed abatement levels. Denoting the outside equilibrium efforts of a single deserter by (e^d, f^d), desertion utility is

$$u_i^d(e_d, f_d) = \underbrace{\left(\frac{f^* - f_i^0}{f^*}\right)}_{=x\in[0,1]} y(e_i^d) - s_i m(e_i^d + (n-1)e^* - f_i^d - (n-1)f^*) - c_e(e_i^d) - c_f(f_i^d) \quad (1)$$

for $i \in \mathcal{N}$ and $n > 2$ (since there must be at least two players left in the agreement after i deserts). A sufficiently large fraction x will successfully deter free-riding on reductive efforts and can be seen as alternative to the blunt global threat represented by contract C'.[3]

A further step in this direction is the formation of an exclusive trade agreement. If a deserter can be excluded from the fraction of trade corresponding to the necessary abatement investments within the agreement, then individual desertion can, again, be discouraged. As above, green production—generated by the expanded abatement effort—is traded among countries and produces wealth. Our model does not take into account the international trading aspects of production and thus there is no direct way of measuring the involved consequences to individual wealth.[4] A simple (ad-hoc) way of nevertheless capturing the idea of enforcement through an exclusive trading agreement is to restrict trade on (and therefore capitalising on) the fraction of productive output corresponding to the reductive investments within the treaty to agreement members. Then player i's desertion utility in (1) can be reduced by using

$$x(\boldsymbol{f}) = \left(1 - \frac{f^* - f_i^0}{e^*}\right) \quad (2)$$

in which reductive effort f_i^0 is the equilibrium abatement level without agreement. As indicated above, to some extent this choice is arbitrary.[5] The intuition for (2) is that an agreement defector j can free-ride on the reductive efforts of agreement members (through a cleaner environment) but is punished by restricted access to the agreement market consisting of the tradables $\sum_{h \neq j} y(e_h)$.

Returning to the simple quadratic cost, square-root production example of section 5 in the main text, this implies that it is individually rational to participate in the agreement if

$$u_i(e^*, f^*) \geq x(\boldsymbol{f}) e_i^{\frac{1}{2}} - \frac{1}{n}\left(e_i + (n-1)e^* - f_i - (n-1)f^*\right)^2 - e_i^2 - f_i^2. \quad (3)$$

Solving the deserter's maximisation problem (on the rhs) for the exclusive trade agreement under (2) leads to the first-order conditions

$$2f^* + \frac{e^* - f^* + f_i^0}{2e^*\sqrt{e_i^d}} = \frac{2(e_i^d + f^* - f_i^d + e^*(n-1) + e_i^d n)}{n}, \; f_i^d = \frac{e_i^d + (e^* - f^*)(n-1)}{n+1} \quad (4)$$

[2] Models on certification and standard settings have been studied intensely; see for instance Lerner & Tirole (2006) or Harbaugh et al. (2011)

[3] To some extent, the particular choice of $x(\boldsymbol{f})$ is arbitrary. Any function of abatement efforts implementing sufficient deterrence (such as the function used for the exclusive trade example below) could be used instead.

[4] Modelling both these aspects formally is possible and certainly provides grounds for future research.

[5] If monitoring of the deserter's abatement efforts is good, then the actual level f_i^d could be used. (In the present example setup, this leads to a rather unintuitive corner solution.) The idea, however, seems important for dealing with competing abatement agreements: As long as they are effective, there is no reason to punish them.

in which (e^*, f^*) are the equilibrium effort levels provided inside the agreement. Plotting the utilities from the desertion efforts (e_i^d, f_i^d) solving above first-order conditions results in a graph similar to figure 2 in the main text for the case of $n = 3$ showing that deviations are not profitable. Thus, the exclusive trade agreement ensures participation. (The analysis is nearly identical for the labelling setup discussed above and therefore not replicated.) As the severity of punishment $(1 - x(\boldsymbol{f}))$ in (2) is given by

$$\frac{f^* - f_i^0}{e^*} = \frac{6 - 3^{1/3}(-2-4n)^{2/3}(-1-n)^{1/3}}{6(1+n)} \tag{5}$$

which is increasing in n, the punishment gets more severe for larger n and participation is easier to obtain in the general case. (The limit as $n \to \infty$ equals $2^{1/3}/3^{2/3} \approx .606$.)

A.2 The choice of ranking technology

Consider a n-player extension of the problem of section 5 in the main text with prize structure $\left(\beta, \frac{1-\beta}{n-1}, \ldots, \frac{1-\beta}{n-1}\right)$. The present example shows that efficiency can also be obtained in proposition 2 for a 'difference-form' success function. The specific properties of the generalised Tullock success function which we use in the remainder of the paper are therefore not crucial to our results. Difference-form success functions have been widely used in the literature, for instance by Che & Gale (2000), but suffer from the lack of a generally accepted, simple extension to more than two players. We define player i's probability of winning as

$$p_i(\Delta) = \frac{\exp^{\Delta_i^r}}{\sum_{j=1}^n \exp^{\Delta_j^r}}, \text{ where } \Delta = (\Delta_1, \ldots, \Delta_n),\ \Delta_i = f_i - \frac{\sum_{j \neq i} f_j}{n-1}, \text{ and } r > 0. \tag{6}$$

Setting $P = (1-\alpha)(e_i^y + (n-1)e_j^y)$, $y \in (0,1)$, $m, b > 1$ and all $j \neq i$ equal, player i's individual problem is to

$$\max_{(e_i, f_i)} \ \alpha e_i^y + p_i(\Delta)\beta P + (1 - p_i(\Delta))\frac{1-\beta}{n-1}P - s_i(e_i + (n-1)e_j - f_i - (n-1)f_j)^m - (e_i^b + f_i^b)$$

which, in symmetric equilibrium $e = e_i = e_j$, $f = f_i = f_j$ gives for any $p_i(\Delta)$

$$\alpha = \frac{e^{-y}\left((e-f)\left(be^b n - e^y y\right) + em((e-f)n)^m s_i\right)}{(e-f)(n-1)y},$$

$$\beta = \frac{e^{-y}\left(-b(e-f)f^b(n-1)n + f\left(m(n-1)((e-f)n)^m s_i + e^y(e-f)n^2(\alpha-1)p_i'(0)\right)\right)}{(e-f)fn^3(\alpha-1)p'(0)}$$

in which $\Delta = 0$ is the equilibrium vector of deviations.

Plugging in the efficient efforts from M:(23), employing (6), and returning to the example setup from section 5 (i.e., $n = 2$, $y = 1/2$, $b = m = 2$, and $s_i = 1/2$), this results in a very similar efficient mechanism as under the Tullock success function

$$\alpha^* = \frac{3}{5},\ \beta^* = \frac{r + (5/6)^{2/3}}{2r} \tag{7}$$

in which $\beta^* \in (.5, 1]$ is ensured for $r \geq (5/6)^{2/3} \approx 0.89$. A picture nearly identical to figure 2 confirms, for instance, $\langle \alpha^*, \beta^*, r = 2 \rangle$ as equilibrium. The precise form of ranking technology employed is thus immaterial to our results.

Appendix B: Proofs

Proof of proposition 1. Efficient efforts are extending M:(2) as the pair (e^*, f^*) solving

$$y'(e) \ = m'(ne - nf) + c_e(e, f),\ m'(ne - nf) = c_f(e, f). \tag{8}$$

Let $P = (1-\alpha)\sum_{h=1}^{n} y(e_h)$. Since we are only interested in deviations from symmetric equilibrium, we set $e_j = e_{-i}$. Rewriting M:(8) for the 2-prize structure $\left(\beta^1, \frac{1-\beta^1}{n-1}, \ldots, \frac{1-\beta^1}{n-1}\right)$ results in

$$\alpha y(e_i) + \beta^1 p_i^1(\boldsymbol{f})P + \sum_{h\ =2}^{n} \frac{1-\beta^1}{n-1} p_i^h(\boldsymbol{f})P - s_i m\left(e_i + (n-1)e_j - f_i - (n-1)f_j\right) - c_e(e_i) - c_f(f_i)$$

which simplifies to

$$\alpha y(e_i) + \beta^1 p_i^1(\boldsymbol{f})P + \frac{1-\beta^1}{n-1}(1 - p_i^1(\boldsymbol{f}))P - s_i m\left(e_i + (n-1)e_j - f_i - (n-1)f_j\right) - c_e(e_i) - c_f(f_i).$$

The symmetric $e = e_i = e_j$, $f = f_i = f_j$, first-order conditions for this problem are

$$\begin{aligned} c_e'(e) + s_i m'(ne - nf) &= \frac{1 - \beta^1 + \alpha(n + \beta^1 - 2) + (1-\alpha)(n\beta^1 - 1)}{n-1} p(f) y'(e) \\ c_f'(f) &= s_i m'((e-f)n) + \frac{n(1-\alpha)(n\beta^1 - 1)}{n-1} p'(f) y(e). \end{aligned} \tag{9}$$

Plugging in (8) and imposing $s_i = 1/n$, one obtains

$$\alpha^* \ = 1 - \frac{y'(e^*) - c_e'(e^*)}{y'(e^*)} \text{ and } \beta^* = \frac{1}{n} + \frac{(n-1)^2 y'(e^*)}{n^3 y(e^*) p'(\boldsymbol{f^*})} \tag{10}$$

which can be achieved for any exogenously given ranking technology $p(\boldsymbol{f^*})$. ☐

Proof of proposition 2. Since under our assumptions M:(8) is fully separable we can split the problem into two independent problems along the respective effort dimensions. Setting $P = (1 - \alpha)\sum_{h=1}^{n} y(e_h)$, the two separate problems are

$$\begin{aligned} \alpha y(e_i) + \beta^1 p_i^1(\boldsymbol{f^*})P + \frac{1-\beta^1}{n-1}(1 - p_i^1(\boldsymbol{f^*}))P - s_i m\left(e_i + (n-1)e^* - n\, f^*\right) - c_e(e_i) - c_f(f^*), \\ \alpha y(e_i^*) + \beta^1 p_i^1(\boldsymbol{f})P + \frac{1-\beta^1}{n-1}(1 - p_i^1(\boldsymbol{f}))P - s_i m\left(ne^* - f_i - (n-1)f^*\right) - c_e(e^*) - c_f(f_i). \end{aligned}$$

1) We show that exerting productive effort $e_i = e^*$ gives a global maximum. As players are symmetric and we are looking for a profitable deviation from the efficient level we set $\boldsymbol{f}^* = (f_1 = f^*, \ldots, f_n = f^*)$ implying that the probability of winning is $p_i^1(\boldsymbol{f^*}) = 1/n$. Thus the problem simplifies to

$$\alpha y(e_i) + \frac{1}{n}P - s_i m\left(e_i + (n-1)e^* - (n)f^*\right) - c_e(e_i) - c_f(f^*) \tag{11}$$

giving the first-order condition for productive effort e_i as[6]

$$\underbrace{y'(e_i)\left(\alpha + \frac{1}{n}(1-\alpha)\right)}_{\searrow} = \underbrace{s_i m'\left(\max\{0, e_i + (n-1)e_j^* - (n)f^*\}\right)}_{\nearrow} + \underbrace{c_{e_i}'(e_i)}_{\nearrow}.$$

[6] It is routine to verify that both first-order conditions identify a maximum.

Notice that output is strictly increasing in e_i and is strictly concave. Thus $y''(e_i) < 0$ and $y'(e_i)$ is decreasing. Both cost functions are increasing and convex, therefore $s_i m''(\cdot) + c''(e_i) > 0$ and the rhs is increasing. As $y'(0) > s_i m'(\max\{0, (n-1)e^* - nf^*\}) + c'(0)$,[7] this confirms single crossing of rhs and lhs and ensures the existence of an equilibrium.

2) We now demonstrate global optimality of $f_i = f^*$. Assuming efficient productive effort provision, the first-order condition for reductive effort is

$$\underbrace{n\, y\,(e^*)\,(1-\alpha)\,(\beta\, n - 1)\, p'(f_i, f^*)}_{=B} = \underbrace{c'_{f_i}(f_i)}_{=C\nearrow} - \underbrace{s_i m'(\max\{0, ne^* - (n-1)f^* - f_i\})}_{=A\searrow}. \quad (12)$$

Notice that the rhs is strictly increasing as we know that, with respect to f_i, $s_i m''(\cdot) \leq 0$ and thus that A is decreasing and the cost function is convex. Without further assumptions on the monitoring technology $p(\cdot)$ we cannot sign the slope of B. Notice, however, that increasing the slope of the (convex) cost function $c'_{f_i}(f_i)$ sufficiently guarantees single crossing and thus a unique maximum whatever the precise specification of $p(\cdot)$.

3) We now show that (12) identifies a global maximum for the Tullock success function.[8] Again, $s_i m(\max\{0, ne^* - (n-1)f^* - f_i\}) > 0$ for $f_i = 0$ while $p'(f_i, f^*) = 0$ and thus the lhs of (12) is zero at $f_i = 0$ while the rhs is negative. Single crossing is immediate for the case of $r \in (0, 1]$ as B is (weakly) decreasing. In the general case of

$$p_i(\boldsymbol{f}) = \frac{f_i^r}{\sum_{j=1}^n f_j^r}, \; r > 1, \quad (13)$$

the function B has a single critical point and is decreasing when $f_i \geq f^* \left(\frac{(n-1)(r-1)}{r+1}\right)^{1/r}$.

To get single crossing if the two curves are increasing we need to ensure either strict concavity or convexity for the lhs and strict convexity for the rhs and prove that if $f_i = 0$, lhs is larger than the rhs. As we have not specified anything about our functions regarding the third derivative we illustrate this point using the specific $c'_{f_i}(f_i) = bf^{b-1}$ and $s_i m(\max\{0, ne^* - (n-1)f^* - f_i\}) = s_i(\max\{0, ne^* - (n-1)f^* - f_i\})^b$. We also set $s_i = \frac{1}{n}$. We find that both curves have an inflection point, thus we need to find a condition to ensure single crossing.

We first show that the rhs starts out negative and eventually becomes positive as for $f_i = 0$ we have $C - A = -s_i m'(\max\{0, ne^* - (n-1)f^*\}) < 0$. Therefore, as long as the lhs is positive and the rhs negative, the two curves cannot cross. We find that $C - A < 0$ for $f_i < f^* \frac{2}{n^{\frac{1}{b-1}} + 1}$ because

$$C - A = f_i^{b-1} b - \frac{(\overbrace{ne^* - (n-1)f^*}^{=2f^*} - f_i)^{b-1} b}{n} = 0 \Leftrightarrow (2f^* - f_i)^{b-1} = n f_i^{b-1}. \quad (14)$$

Moreover, for the rhs, the inflection point occurs when the curve is negative, and it is first concave and then convex. Thus we can conclude than when the curve is above zero, it is strictly convex.

[7] Since output is concave and the sum of cost functions is convex in e_i, the above inequality holds.

[8] A nearly identical argument can be made for any other ratio-based success function. In that more general case, however, we cannot derive an explicit existence threshold.

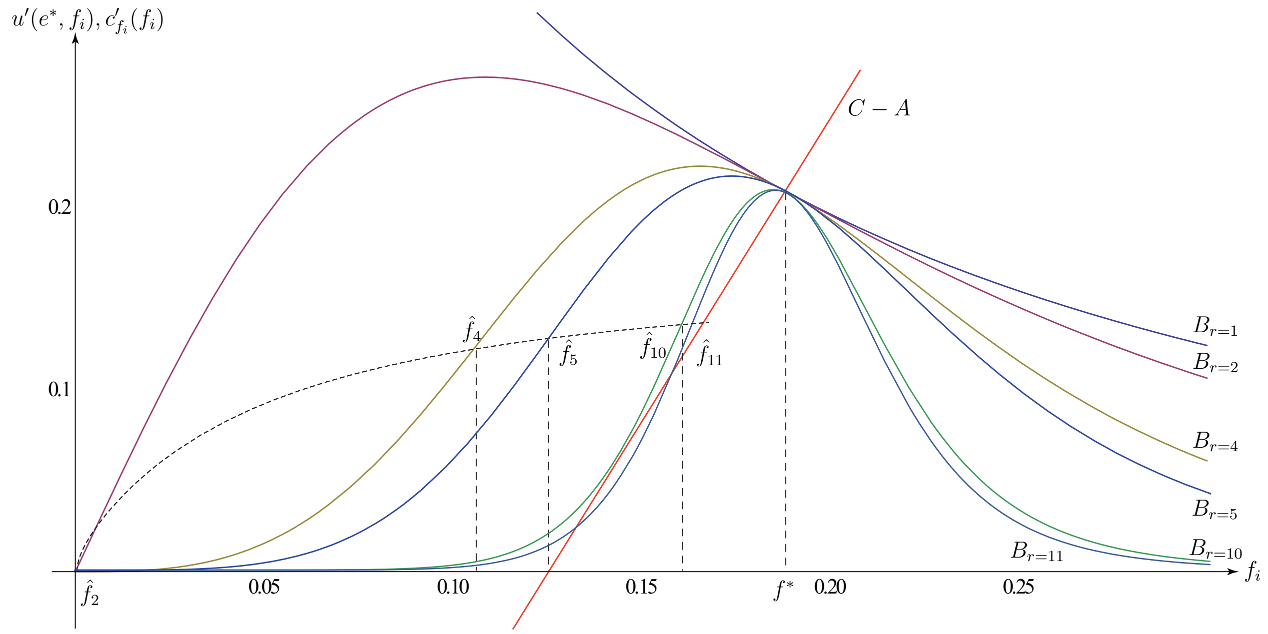

Figure 1: Single crossing in equation (12) ensures a unique global maximum at $f_i = f^*$ for the example setup of section 5. The dotted line gives the location of inflection points $\hat{f}$ for different r.

We find that $(C-A)'' < 0$ for $f_i < f^* \frac{2}{n^{\frac{1}{b-3}}+1}$ and $f_i < f^* \frac{2}{n^{\frac{1}{b-3}}+1} < f^* \frac{2}{n^{\frac{1}{b-1}}+1}$ because[9]

$$(C-A)'' = f_i^{b-3}(b-2)(b-1)b - \frac{(2f^*-f_i)^{b-3}(b-2)(b-1)b}{n} = 0 \;\Leftrightarrow (2f^*-f_i)^{b-3} = n f_i^{b-3},$$

$$\Leftrightarrow f^* \frac{2}{n^{\frac{1}{b-1}}+1} - f^* \frac{2}{n^{\frac{1}{b-3}}+1} = 2 \frac{f^*\left(n^{\frac{1}{b-3}} - n^{\frac{1}{b-1}}\right)}{\left(n^{\frac{1}{b-1}}+1\right)\left(n^{\frac{1}{b-3}}+1\right)} \geq 0.$$

We conclude that the rhs is strictly increasing and convex when it is positive.

For the lhs, there are two inflection points: one in the increasing part and the other in the decreasing part. In the increasing part we find a condition which implies that the inflection occurs if the rhs is negative.[10] A sufficient condition for single-peakedness is therefore that

$$\frac{2^r (f^*)^r}{\left(n^{\frac{1}{b-1}}+1\right)^r} \geq \underbrace{\frac{(n-1)\left(2(f^*)^r(r^2-1) - \sqrt{3}\sqrt{(f^*)^{2r}r^2(r^2-1)}\right)}{2+3r+r^2}}_{=:\hat{f}}. \tag{15}$$

Thus if the rhs of (12) is positive, it is also strictly convex. If (15) is respected, the lhs is strictly concave or convex. Notice also that at the inflection point, the rhs is positive and the lhs is negative and therefore the lhs is larger than the rhs. The geometric intuition of (15) is shown in figure 1 for the setup of the example section 5 in the main text. The figure shows a family of curves B for $r \in \{1, 2, 4, 10, 11\}$ with inflection points labelled $\hat{f}_2$, $\hat{f}_4$, $\hat{f}_{10}$, and $\hat{f}_{11}$, respectively. Condition (15) is fulfilled as long as the red cost curve $C-A$ is negative at the respective inflection point. This is true for $r = \{2, 4\}$ but (15) fails for higher r. Uniqueness, however, is only lost for $r > 10$. □

[9] This is true for any $b \geq 3$.

[10] The inflection point in the decreasing part does not matter. As long as one curve is increasing and the other is decreasing they can only cross once.

Proof of proposition 3. Player i's equilibrium participation utility for $P = (1-\alpha)ny(e^*)$ under the efficiency-inducing contract $C = \langle \alpha^*, \beta^*; p^*(\boldsymbol{f})\rangle$ defined in (8) with prizes $\left(\beta, \frac{1-\beta}{n-1}, \dots, \frac{1-\beta}{n-1}\right)$ is

$$\begin{aligned} u_i(e^*, f^*) &= \alpha^* y(e^*) + \tfrac{1}{n}\beta^* P + (n-1)\tfrac{1}{n}\tfrac{1-\beta^*}{n-1}P - s_i m\left(ne^* - nf^*\right) - c_e(e^*) - c_f(f^*) \\ &= \alpha y(e^*) + \tfrac{1}{n}(1-\alpha^*)ny(e^*) - s_i m\left(ne^* - nf^*\right) - c_e(e^*) - c_f(f^*) \\ &= y(e^*) - s_i m\left(ne^* - nf^*\right) - c_e(e^*) - c_f(f^*). \end{aligned} \tag{16}$$

Now consider the contract $C' = \langle \alpha', \beta'; p'\rangle$. Since second-stage efforts $e(\alpha', \beta', p'), f(\alpha', \beta', p')$ are continuous in α', it is sufficient to consider the extreme case of $\alpha' = 1$ which implements no contest at all. A deserter's utility when subjected to C' can therefore be driven down to[11]

$$u_i^s(e_i^s, f_i^s) = y(e_i^s) - s_i m(e_i^s + (n-1)e^a - f_i^s - (n-1)f^a) - c_e(e_i^s) - c_f(f_i^s) \tag{17}$$

in which e^a and f^a are the equilibrium inside agreement efforts prescribed by C'. For $\alpha' = 1$, these equal the equilibrium efforts without agreement e_i^s, f_i^s. Hence (16) and (17) are identical but implement different efforts. Since both $e^s > e^*$ and $f^s < f^*$, it is therefore individually rational to join the agreement implementing C if the alternative is C' because the cost differential

$$s_i(m(ne^s - nf^s) - m\left(ne^* - nf^*\right)) \tag{18}$$

is increasing in n and convex. As the efficient allocation is welfare maximising, this outweighs any productivity gains from free-riding $y(e^s) - y(e^*) + c_e(e^*) + c_f(f^*) - c_e(e^s) - c_f(f^s)$. Thus, every player finds it individually rational to join the reductive contest if threatened by alternative C'. □

Proof of proposition 4. From (12), we know that

$$\alpha_i = \frac{y_i'(e_i)(1-H_i) - (1-s_i)m'\left(G\right)}{y_i'(e_i)(1-H_i)}, \text{ in which } H_i = \beta_i p_i^1(\boldsymbol{f}) + \frac{\sum_{j\neq i}(1-\beta_j)p_j^1(\boldsymbol{f})}{(n-1)} \tag{19}$$

$$\beta_i = \frac{(n-1)\left((1-s_i)m'\left(G\right)\right) - \sum_{j\neq i}(1-\beta_j)p_{j(f_i)}^1(\boldsymbol{f})P}{p_{i(f_i)}^1(\boldsymbol{f})(n-1)P}. \tag{20}$$

Equation (20) can be rewritten as

$$p_{i(f_i)}^1(\boldsymbol{f})\beta_i = \frac{(1-s_i)m'(G)}{P} - \sum_{j\neq i} p_{j(f_i)}^1(\boldsymbol{f})\frac{1-\beta_j}{n-1}. \tag{21}$$

Integrating over f_i, it follows that

$$p_i^1(\boldsymbol{f})\beta_i = \frac{(1-s_i)m(G)}{P} - \sum_{j\neq i} p_j^1(\boldsymbol{f})\frac{1-\beta_j}{n-1} \tag{22}$$

as $m'(G) = c_{f_i}'(f_i^*)$. Equation (22) in H_i leads to $1 - H_i = \dfrac{P - (1-s_i)m'(G)}{P}$ which we insert into (19) to obtain

$$\alpha_i = 1 - \frac{(1-s_i)m'(G)P}{y_i'(e_i)[P - (1-s_i)m(G)]} \quad \forall i. \tag{23}$$

[11] Notice that the latter formulation requires $n > 2$ as the contest can only produce incentives if at least two players participate in the contest.

As $P = \sum_{j=1}^n y_j(e_j)(1-\alpha_j)$, $(\alpha_1, \ldots, \alpha_n)$ are described by a system of n equations independent of $(\beta_1, \ldots, \beta_n)$ given by

$$\begin{aligned}(1-\alpha_i)^2 y_i'(e_i) y_i(e_i) + (1-\alpha_i) y_i'(e_i) \sum_{j\neq i} y_j(e_j)(1-\alpha_j) \\ - (1-s_i)(1-\alpha_i) m(G) y_i'(e_i) + (1-s_i) m'(G) \sum_{j=1}^n y_j(e_j)(1-\alpha_j) = 0.\end{aligned} \tag{24}$$

From a fixed point argument such as, for instance, Brouwer's theorem, the vector $(\alpha_1, \ldots, \alpha_n)$ exists. Then (20) leads do a system of linear equations and $(\beta_1, \ldots, \beta_n)$ exists as well. □

Proof of proposition 5. The variance of the pooled, n-player redistributed resource is

$$\begin{aligned} & \mathbb{V}\left[a\frac{y(e_1,\varepsilon_1)+\cdots+y(e_n,\varepsilon_n)}{n} + by(e_i,\varepsilon_i)\right] \\ = & \; a^2\left(\frac{1}{n^2}\mathbb{V}(\varepsilon_1)+\cdots+\frac{1}{n^2}\mathbb{V}(\varepsilon_n)\right) + b^2\,\mathbb{V}(\varepsilon_i) + 2\frac{a}{n}b\,\mathrm{Cov}(\varepsilon_i,\varepsilon_i) \\ = & \; a^2\frac{n}{n^2}\sigma^2 + b^2\sigma^2 + \frac{2}{n}ab\sigma^2 \\ = & \; \sigma^2\frac{a^2+2ab+nb^2}{n}\end{aligned} \tag{25}$$

for arbitrary constants a, b and any player $i \in \mathcal{N}$. Individual equilibrium contest payoff is the compound lottery

$$L = \begin{cases} \dfrac{1}{n}: & \beta^* \displaystyle\sum_{j=1}^n (1-\alpha^*) y(e^*,\varepsilon_j) + \alpha^* y(e^*,\varepsilon_i) \\ 1-\dfrac{1}{n}: & \dfrac{1-\beta^*}{n-1} \displaystyle\sum_{j=1}^n (1-\alpha^*) y(e^*,\varepsilon_j) + \alpha^* y(e^*,\varepsilon_i). \end{cases} \tag{26}$$

Hence, the parameters a and b in (25) are

$$a = \beta^*(1-\alpha^*) + (n-1)\frac{1-\beta^*}{n-1}(1-\alpha^*) = 1-\alpha^*, \;\; b = \alpha^* \tag{27}$$

which, when plugged into the last expression in (25), imply that the variance from the pooled output shocks alone is

$$\mathbb{V}_\varepsilon(\mathbb{E}(L|\varepsilon)) \;\; = \;\; \sigma^2\frac{1+(\alpha^*)^2(n-1)}{n} < \sigma^2. \tag{28}$$

To this variance stemming from the individual shocks ε_i alone we have to add the variance introduced through the prize structure of the contest itself. The expectation of the contest payoff is

$$\mathbb{E}[L|\varepsilon] = \frac{1}{n}(1-\alpha^*)\beta^* n y(e^*,\varepsilon) + \left(1-\frac{1}{n}\right)\frac{(1-\beta^*)}{n-1}(1-\alpha^*) n y(e^*,\varepsilon) + \alpha^* y(e^*,\varepsilon) = y(e^*,\varepsilon) \tag{29}$$

and, hence, $\mathbb{E}[L|\varepsilon] = y(e,\varepsilon)$ in which we drop the subscript on the individual shock because they are i.i.d.; the contest variance is therefore

$$\begin{aligned}\mathbb{V}(L|\varepsilon) \;\; = \;\; & \frac{1}{n}((1-\alpha)\beta^* n y(e,\varepsilon) + \alpha^* y(e,\varepsilon) - \mathbb{E}[L|\varepsilon])^2 + \\ & \frac{1}{n}((1-\alpha^*)(1-\beta^*) n y(e,\varepsilon) + \alpha^* y(e,\varepsilon) - \mathbb{E}[L|\varepsilon])^2 \\ = \;\; & \frac{(1-\alpha^*)^2(n((2(\beta^*-1)\beta^*+1)n-2)+2)}{n}\,\mathbb{E}[y(e,\varepsilon)^2].\end{aligned} \tag{30}$$

Pulling (30) together with (28) gives, by the law of total variance $\mathbb{V}[L] = \mathbb{E}[\mathbb{V}[L|\varepsilon]] + \mathbb{V}[\mathbb{E}[L|\varepsilon]]$, the agreement member's payoff variance

$$\frac{\sigma^2\left((\alpha^*)^2(n-1)+1\right)}{n} + \mathbb{E}[y(e,\varepsilon)^2]\frac{(\alpha^*-1)^2(n((2(\beta^*-1)\beta^*+1)n-2)+2)}{n} \tag{31}$$

which we need to be smaller than standalone income variance σ^2. This condition holds if[12]

$$\mathbb{E}[y(e,\varepsilon)^2]\frac{(1-\alpha^*)(n((2(\beta^*-1)\beta^*+1)n-2)+2)}{(\alpha^*+1)(n-1)} \leq \sigma^2. \tag{33}$$

Into this general threshold expression, we now substitute the equilibrium parameters of our standard square-root production $y = \sqrt{e}$ and quadratic costs example for n players

$$\begin{aligned} e^* &= \frac{\left(\frac{n+1}{2n+1}\right)^{2/3}}{2\sqrt[3]{2}}, & f^* &= \frac{n}{2\sqrt[3]{2}\sqrt[3]{(n+1)(2n+1)^2}}, \\ \alpha^* &= \frac{4\sqrt{e^*}n(2e^*-f^*)-1}{n-1}, & \beta^* &= \frac{8(e^*)^2r+2f^*(n-1)(e^*-2f^*)-4e^*f^*r-\sqrt{e^*}r}{8(e^*)^2nr-4e^*f^*nr-\sqrt{e^*}nr} \end{aligned}$$

translates (33) into the condition

$$\sigma^2 \geq \frac{n\left(-2\left(-n^2+n+2\right)^2r^2-(n-1)^2n^2+2(n-2)(n-1)(n+1)nr\right)+\sqrt[3]{-n-\frac{1}{2}}}{4(-n-1)^{4/3}(n-1)(n(6n+7)+2)r^2} \tag{34}$$

which represents the threshold beyond which individual variance σ^2 can be compressed in the contest-based redistributive agreement. □

Proof of proposition 6. Only the variance stemming form the shocks (28) changes when we move to the non-i.i.d. case relative to the previous proof; condition (30) remains unchanged. The variance of the redistributed resource is now for arbitrary player i

$$\begin{aligned} & \mathbb{V}\left[(a/n)(y(e_1,\varepsilon_1)+\cdots+y(e_n,\varepsilon_n))+by(e_i,\varepsilon_i)\right] \\ =\; & na^2\,\mathbb{V}[\varepsilon_i]+(n^2-n)a^2\operatorname{Cov}(\varepsilon_i,\varepsilon_j)+ab\,\mathbb{V}[\varepsilon_i]+(n-1)ab\operatorname{Cov}(\varepsilon_i,\varepsilon_j) \\ =\; & \alpha^*\frac{\operatorname{Cov}(\varepsilon_i,\varepsilon_j)(n-1)+\mathbb{V}[\varepsilon_i]}{n} \\ =\; & \alpha^*\frac{\bar{\sigma}^2(n-1)+\sigma^2}{n} \end{aligned} \tag{35}$$

in which we set $a = \alpha^*$ and $b = 1-\alpha^*$ as determined in (27). Adding the variance from the contest itself (30) as in the previous proof yields the individual equilibrium contest payoff variance

$$\alpha^*\frac{\bar{\sigma}^2(n-1)+\sigma^2}{n} + \mathbb{E}[y(e,\varepsilon)^2]\frac{(\alpha^*-1)^2(n((2(\beta^*-1)\beta^*+1)n-2)+2)}{n} \tag{36}$$

which we need to be smaller than standalone income variance σ^2. This is the case if

$$\sigma^2 \geq \frac{\bar{\sigma}^2(n-1)\alpha^*+\mathbb{E}[y(e,\varepsilon)^2](1-\alpha^*)^2(2+n(n(1+2(\beta^*-1)\beta^*)-2))}{n-\alpha^*}. \tag{37}$$

[12] Since both the variance and output magnitudes depend on the chosen measurement units, it is perhaps preferable to transform (33) into the general, scaling-invariant "signal to noise ratio" requirement that

$$\frac{\mathbb{E}[y^2(e,\varepsilon)]}{\sigma^2}\begin{cases} \leq \dfrac{(\alpha^*+1)(n-1)}{(1-\alpha^*)(n((2(\beta^*-1)\beta^*+1)n-2)+2)} & \text{if } \sigma^2 \geq 1, \\ > \dfrac{(\alpha^*+1)(n-1)}{(1-\alpha^*)(n((2(\beta^*-1)\beta^*+1)n-2)+2)} & \text{if } \sigma^2 < 1. \end{cases} \tag{32}$$

Substituting our standard example parameters gives

$$\sigma^2 \geq \frac{\bar{\sigma}^2(n-1)(n+1)}{2n^2-1} + \frac{n^2\,\mathbb{E}[y(e,\varepsilon)^2]\left(2\left(-n^2+n+2\right)^2 r^2 + (n-1)^2 n^2 - 2(n-2)(n-1)(n+1)nr\right)}{2(n+1)^2(2n+1)\left(2n^2-1\right)r^2} \tag{38}$$

which is equal to the condition $\sigma^2 \geq \frac{3}{7}\bar{\sigma}^2 + \frac{8}{315r^2}\,\mathbb{E}[y(e,\varepsilon)^2]$ for the case of two players which again implies that the agreement compresses variance if the standalone variance exceeds $.0018 + .43\bar{\sigma}^2$. $\square$

Proof of proposition 7. Since each individual output $y(e_i, \varepsilon_i)$ has variance σ^2 and covariance of two distinct variables σ_{ij}, the variance of the prize pool is (for equilibrium α)

$$\begin{aligned}
\mathbb{V}[u_i(\boldsymbol{e},\boldsymbol{f};\boldsymbol{\varepsilon})] &= \mathbb{V}[\alpha y(e_i,\varepsilon_i)] + \mathbb{V}\left[(1-\alpha)\frac{1}{n}\sum_{i=1}^n y(e_i,\varepsilon_i)\right] + 2\alpha(1-\alpha)\frac{1}{n}\operatorname{Cov}\left(y(e_i,\varepsilon_i), \sum_{i=1}^n y(e_i,\varepsilon_i)\right)\\
&= \alpha^2\,\mathbb{V}[y(e_i,\varepsilon_i)] + (1-\alpha)^2\frac{1}{n^2}\sum_{i=1}^n \mathbb{V}\left[y(e_i,\varepsilon_i)\right] + 2\alpha(1-\alpha)\frac{1}{n}\operatorname{Cov}\left(y(e_i,\varepsilon_i), \sum_{i=1}^n y(e_i,\varepsilon_i)\right)\\
&+ 2\frac{(1-\alpha)^2}{n^2}\operatorname{Cov}\left(\sum_{i=1}^n y(e_i,\varepsilon_i), \sum_{j=1}^n y(e_j+\varepsilon_j)\right)\\
&= \sigma^2\left[\alpha^2 + (1-\alpha)^2\frac{1}{n}\right] + \alpha(1-\alpha)\frac{1}{n}\sum_{i\neq j}\sigma_{ij} + 2\frac{(1-\alpha)^2}{n}\sum_{1\leq i<j\leq n}\sigma_{ij}\\
&\leq \sigma^2\left[\alpha^2 + (1-\alpha)^2\frac{1}{n}\right] + 2\alpha(1-\alpha)\frac{n-1}{n}\sigma^2 + \frac{(1-\alpha)^2}{n}n(n-1)\sigma^2\\
&= \frac{\sigma^2}{n}\left[2\alpha^2 - 2\alpha + n\right]\\
&< \sigma^2 \quad \forall \alpha \in (0,1).
\end{aligned}$$

$\square$

Proof of proposition 8. Let us first consider independent shocks. For equilibrium α_i, β_i,

$$\begin{aligned}
\mathbb{V}[u_i(\boldsymbol{e}^*,\boldsymbol{f}^*);\boldsymbol{\varepsilon}] &= \mathbb{V}[\alpha_i y(e_i^*,\varepsilon_i) + p_i(\boldsymbol{e}^*,\boldsymbol{f}^*)\beta_i[(1-\alpha_i)y(e_i^*,\varepsilon_i)\\
&\quad + (1-\alpha_j)y(e_j^*,\varepsilon_j)] + (1-p_i(\boldsymbol{e}^*,\boldsymbol{f}^*))(1-\beta_j)[(1-\alpha_i)y(e_i^*,\varepsilon_i) + (1-\alpha_j)y(e_j^*,\varepsilon_j)]\\
&= [\alpha_i + (p_i(\boldsymbol{e}^*,\boldsymbol{f}^*)\beta_i + (1-p_i(\boldsymbol{e}^*,\boldsymbol{f}^*))(1-\beta_j))(1-\alpha_i)]^2\sigma^2\\
&\quad + [p_i(\boldsymbol{e}^*,\boldsymbol{f}^*)\beta_i + (1-p_i(\boldsymbol{e}^*,\boldsymbol{f}^*))(1-\beta_j)]^2(1-\alpha_j)^2\sigma^2\\
&= \alpha_i^2\sigma^2 + 2\alpha_i(1-\alpha_i)[p_i(\boldsymbol{e}^*,\boldsymbol{f}^*)\beta_i + (1-p_i(\boldsymbol{e}^*,\boldsymbol{f}^*))(1-\beta_j)]\sigma^2\\
&\quad + [p_i(\boldsymbol{e}^*,\boldsymbol{f}^*)\beta_i + (1-p_i(\boldsymbol{e}^*,\boldsymbol{f}^*))(1-\beta_j)]^2((1-\alpha_i)^2 + (1-\alpha_j)^2)\sigma^2\\
&\leq \alpha_i^2\sigma^2 + 2\alpha_i(1-\alpha_i)\beta_i\sigma^2 + ((1-\alpha_i)^2 + (1-\alpha_j)^2)[p_i(\boldsymbol{e}^*,\boldsymbol{f}^*)^2(-1+\beta_i+\beta_j)^2\\
&\quad + 2p_i(\boldsymbol{e}^*,\boldsymbol{f}^*)(1-\beta_j)(-1+\beta_i+\beta_j) + (1-\beta_j)^2]\sigma^2\\
&\leq \{\alpha_i^2 + 2\alpha_i(1-\alpha_i)\beta_i + ((1-\alpha_i)^2 + (1-\alpha_j)^2)\beta_i^2\}\sigma^2.
\end{aligned}$$

Then, $\mathbb{V}[u_i(\boldsymbol{e}^*,\boldsymbol{f}^*);\boldsymbol{\varepsilon}] < \sigma^2$ for all i if and only if

$$1 > \beta_i^2(1-\alpha_j)^2 + (\beta_i - \alpha_i\beta_i - \alpha_i)^2 + 4\alpha_i\beta_i(1-\alpha_i). \tag{39}$$

Under non-independent shocks, the covariance term has to be added

$$
\begin{aligned}
&\mathrm{Cov}([\alpha_i + A(1-\alpha_i)]y(e_i^*, \varepsilon_i), A(1-\alpha_j)y(e_j^*, \varepsilon_j)) \\
&= [\alpha_i + A(1-\alpha_i)]A(1-\alpha_j)\,\mathrm{Cov}[y(e_i^*, \varepsilon_i), y(e_j^*, \varepsilon_j)] && (40)\\
&\leq [\alpha_i + \beta_i(1-\alpha_i)]\beta_i(1-\alpha_j)\sigma_{ij} && (41)\\
&\leq [\alpha_i + \beta_i(1-\alpha_i)]\beta_i(1-\alpha_j)\sigma^2
\end{aligned}
$$

with $A = [p_i(\boldsymbol{e}^*, \boldsymbol{f}^*)\beta_i + (1 - p_i(\boldsymbol{e}^*, \boldsymbol{f}^*))(1-\beta_j)]$. As for the independent shocks, we use the fact $A \leq \beta_i$ (equations (40) & (41)). Then, using the condition (39) for independent shocks we get

$$1 > \beta_i(3 - 2\alpha_i - \alpha_j)(2\alpha_i + \beta_i - \beta_i\alpha_j) + (\beta_i - \alpha_i\beta_i - \alpha_i)^2 \tag{42}$$

which establishes our claim. □

Proof of proposition 9. Assume that M:(22) holds (otherwise we are done), where $u^{\mathsf{rm}}(\cdot)$ and $u^{\mathsf{fr}}(\cdot)$ are both linear functions in their respective arguments $\varepsilon^{\mathsf{rm}}$ and $\varepsilon^{\mathsf{fr}}$. From theorems 1 and 2, it follows that $\varepsilon^{\mathsf{fr}} \leq_{\mathsf{icv}} \varepsilon^{\mathsf{rm}}$ such as $\mathbb{E}[v(\varepsilon^{\mathsf{fr}})] \leq \mathbb{E}[v(\varepsilon^{\mathsf{rm}})]$ for all increasing and concave functions v. Consequently, there exists a sufficiently concave, increasing function v such that $\mathbb{E}[v(u^{\mathsf{rm}}(\boldsymbol{e}^*, \boldsymbol{f}^*; \varepsilon^{\mathsf{rm}}))] \geq \mathbb{E}[v(u^{\mathsf{fr}}(\tilde{\boldsymbol{e}}, \tilde{\boldsymbol{f}}; \varepsilon^{\mathsf{fr}}))]$. □

Appendix C: Stochastic Dominance Relations

This appendix provides analytical tools and new results on stochastic dominance which are useful to establish proposition 9.

Definition 1 (Shaked & Shanthikumar (1994))**.** *Let X and Y be two random variables. X is said to be smaller than Y in the increasing concave order and denoted $X \leq_{\mathsf{icv}} Y$ if and only if*

$$\mathbb{E}[v(X)] \leq \mathbb{E}[v(Y)] \tag{43}$$

for all increasing, concave functions $v(\cdot)$.

In the economics literature, increasing concave orders are typically referred to as *second order stochastic dominance*. Similarly, increasing concave orders with equal means are usually called *mean preserving spreads*. An alternative, equivalent definition is:

Definition 2 (Shaked & Shanthikumar (1994))**.** *Let X and Y be two random variables with F_X and F_Y their continuous cumulative functions. $X \leq_{\mathsf{icv}} Y$ if and only if*

$$\int_{-\infty}^{z} F_X(t)dt \geq \int_{-\infty}^{z} F_Y(t)dt. \tag{44}$$

The following result generalises theorem 5 of Müller (2001) for univariate elliptical distributions and increasing concave orders. The proof is provided for completeness and is an adaptation of that in Müller (2001).

Theorem 1. *Let $X \sim \mathcal{L}(\mu_x, \sigma_x^2)$ and $Y \sim \mathcal{L}(\mu_y, \sigma_y^2)$, which are two elliptically distributed random variables. If $\sigma_y^2 \leq \sigma_x^2$ and $\mu_y \leq \mu_x$ then $X \leq_{\text{icv}} Y$.*

Proof. Given the properties of elliptical distributions, the distribution of X is equal to that of $Y+Z$ with $Z \sim \mathcal{L}(\mu_x - \mu_y, \sigma_x^2 - \sigma_y^2)$.[13] Then,

$$\begin{aligned} \mathbb{E}[v(X)] &= \mathbb{E}[v(Y+Z)] \\ &= \mathbb{E}[\mathbb{E}[v(Y+Z)/Y]] \qquad (45) \\ &\leq \mathbb{E}[v(Y+\mathbb{E}[Z])] \qquad (46) \\ &\leq \mathbb{E}[v(Y)]. \qquad (47) \end{aligned}$$

The Jensen inequality for concave functions leads to (46). Then, using $\mathbb{E}[Z] \leq 0$ (47) follows. □

Theorem 2. *Let $X \sim \mathcal{L}(\mu_x, \sigma_x^2)$ and $Y \sim \mathcal{L}(\mu_y, \sigma_y^2)$, which are two elliptically distributed random variables such that $\mu_y = \mu_x = 0$. Then the following statements are equivalent: (i) $X \leq_{\text{icv}} Y$, and (ii) $\sigma_y^2 \leq \sigma_x^2$.*

Proof. Every probability density $(F_i)'$ is symmetric and continuous. Zero mean implies that the symmetry of the probability density is with respect to the ordinate. Then, every cumulative distribution has an inflection point at zero. Furthermore, X has a probability density with bigger tails than Y because of the ranking of the variance. Consequently of the tails and the inflection point, the cumulative distribution of X is above (below) the one of Y for all value inferior (superior) to zero such that the surfaces between the two cumulative distributions before and after zero are equal. Then,

$$\int_{-\infty}^{z} F_X(t)dt \geq \int_{-\infty}^{z} F_Y(t)dt \qquad (48)$$

which implies that $X \leq_{\text{icv}} Y$. Notice that this implication is true by application of Theorem 1.

Moreover, it is well-known that if $X \leq_{\text{icv}} Y$ and $\mathbb{E}[X] = \mathbb{E}[Y]$ then $\sigma_y^2 \leq \sigma_x^2$ follows (see, for instance, Rothschild & Stiglitz (1970)). □

Appendix D: Further examples

Section 5 of the main paper illustrates the intuition of our principal result: the characterisation of efficient equilibria. This section provides a sequence of further examples for the other results of the paper in a similar environment.

To get a feeling for the magnitudes implied by our example mechanism we plug in the 2011 global GDP of \$80tr, or, among two identical players, \$40tr GDP per player. Thus, our proposed mechanism collects $P = (2/5)80 = \$24.4$tr or \$16tr from each player. The following table lists the redistribution implied by the efficient mechanism. Depending on the precision of monitoring r, the mechanism parameters and transfers are

[13] See Fang et al. (1987) or Landsman & Valdez (2003) for details.

r	β	$1-\beta$	1st	2nd	transfer	%
1	$^2/_3$	$^1/_3$	\$21.3tr	\$10.7tr	±\$5.3tr	±13.3%
2	$^7/_{12}$	$^5/_{12}$	\$18.7tr	\$13.2tr	±\$2.7tr	±6.7%
.	...		...		...	
5	$^8/_{15}$	$^7/_{15}$	\$17.0tr	\$15.0tr	±\$1.0tr	±2.7%
.	...		...		...	
11	$^{17}/_{33}$	$^{16}/_{33}$	\$16.5tr	\$15.5tr	±\$0.5tr	±1.2%

where the rightmost column's percentages are taken from the total symmetric \$40tr GDP. Our sufficient existence condition derived in proposition 2 guarantees existence up to and including $r = 5$. Actual existence, however, is only lost for $r > 11$. At this monitoring level, a transfer of 1.2% of GDP is sufficient to implement both efficient abatement and efficient production.[14]

As pointed out in the model section and the discussion of the sharing mechanism in section 4.2.2, we can alternatively employ the interpretation of where each player gets exactly a contractible 'share' $p_i(\boldsymbol{f})$ of agreement output. Under this mathematically equivalent interpretation, while the incentives incorporated in the mechanism ensure efficient efforts along both dimensions, all symmetric equilibrium transfers cancel out.

D.1 Example of the asymmetric contest

As an illustration of the asymmetric modelling idea of section 3.3 in the main text, the following example provides an analytical characterisation of the efficient equilibrium. Consider the following three-players concretisation of M:(10)[15]

$$\begin{aligned}
&\alpha_i \underbrace{(e_i)}_{=y_i(e_i)} + \Bigg(\underbrace{\frac{f_i^r}{f_1^r + f_2^r + f_3^r}}_{=p_i^1(\boldsymbol{f})} \beta_i + \underbrace{\frac{f_2^r}{f_1^r + f_2^r + f_3^r}}_{=p_2^1(\boldsymbol{f})} \frac{1-\beta_2}{2} + \underbrace{\frac{f_3^r}{f_1^r + f_2^r + f_3^r}}_{=p_3^1(\boldsymbol{f})} \frac{1-\beta_3}{2} \Bigg) P \\
&- s_i \underbrace{(G)^2}_{=m(G)} - \underbrace{\gamma_i \frac{e_i^2}{2}}_{=c_{i,e}(e_i)} - \underbrace{\delta_i \frac{f_i^2}{2}}_{=c_{i,f}(f_i)}
\end{aligned} \tag{49}$$

in which $P = (1-\alpha_1)(e_1) + (1-\alpha_2)(e_2) + (1-\alpha_3)(e_3)$ and $\gamma_i, \delta_i > 0$.[16]

We now derive the asymmetric tax rates α_i^* and redistributive shares β_i^*, $i = 1, 2, 3$, which induce fully efficient productive and abatement equilibrium efforts in the asymmetric contest (49) while balancing the agreement's budget on and off the equilibrium path. The first-order, necessary

[14] The European Union plans to dedicate up to 1/5 of its budget to the fight against global warming. This will represent up to 180 billions euros of climate related project spending over a seven-year period. For details see `http://ec.europa.eu/clima/policies/budget/index_en.htm`.

[15] The two players asymmetric case in the example setup is rather special since $\beta_1 + (1-\beta_2) = 1$ implies $\beta_1 = \beta_2$. Hence, in order to lend itself to a solution, the two players case requires success function slopes of precisely

$$\frac{\partial}{\partial f_i} p_i(\boldsymbol{f}) = \frac{(1-s_i)c_f'(f_i)}{(\beta_1 + \beta_2 - 1)P}.$$

Cases with higher numbers of players than two do not imply equal prize shares and do not induce complications.

[16] We use cost heterogeneity in our development of the asymmetric case. Since only the equalisation of marginal benefits and costs matter for the argument, the asymmetries can be equally thought of on the production side.

condition for (49) with respect to e_i is

$$\alpha_i = \frac{(\alpha_i - 1)\left(2\beta_i f_i^r + \left(\sum_{j\neq i} f_j^r (1-\beta_j)\right)\right)}{2\left(f_1^r + f_2^r + f_3^r\right)} + 2s_i G + \gamma_i e_i, \tag{50}$$

the first-order condition for (49) with respect to f_i is

$$2s_i G = \frac{r f_i^{r-1}\left(\sum_{i=1}^{3} e_i(\alpha_i - 1)\right)\left(\sum_{i=1}^{3} f_j^r \left(2\beta_i - \beta_j - 1\right)\right)}{2\left(f_1^r + f_2^r + f_3^r\right)^2} + \delta_i f_i \tag{51}$$

for $i = 1, 2, 3$. Setting efforts equal to the efficient quantities M:(11) from the planner's problem

$$\begin{aligned} e_i^* &= \frac{\left(\prod_{j\neq i}\gamma_j\right)\left(2\left(\prod_{j\neq i}\delta_j\right) + \delta_i\left(2\left(\left(\sum_{j\neq i}\delta_j\right) + \prod_{j\neq i}\delta_j\right)\right)\right)}{\gamma_1(\gamma_2(\gamma_3(\delta_1(\delta_2(\delta_3+2)+2\delta_3)+2\delta_2\delta_3)+2\delta_1\delta_2\delta_3)+2\gamma_3\delta_1\delta_2\delta_3)+2\gamma_2\gamma_3\delta_1\delta_2\delta_3}, \\ f_i^* &= \frac{2\left(\left(\prod_{j\neq i}\gamma_j\right) + \gamma_i\left(\sum_{j\neq i}\gamma_j\right)\right)\left(\prod_{j\neq i}\delta_j\right)}{\gamma_1(\gamma_2(\gamma_3(\delta_1(\delta_2(\delta_3+2)+2\delta_3)+2\delta_2\delta_3)+2\delta_1\delta_2\delta_3)+2\gamma_3\delta_1\delta_2\delta_3)+2\gamma_2\gamma_3\delta_1\delta_2\delta_3}. \end{aligned} \tag{52}$$

We solve these for the parametrisation of $\delta_2 = \delta_3 = 1$, equal shares $s_i = 1/3$, and the case of $r = 1$ in order to avoid equilibrium existence problems. Further generalisations of this contest can typically only be solved numerically. This yields the mechanism

$$\begin{aligned} \alpha_1^* \ &= \frac{1}{3\gamma_2\gamma_3(4\delta_1-1)(5\delta_1+2)(\gamma_1(\gamma_2(5\gamma_3\delta_1+2\gamma_3+2\delta_1)+2\gamma_3\delta_1)+2\gamma_2\gamma_3\delta_1)} \\ &\Bigg(2\gamma_1^2(4\delta_1-1)\left(9\delta_1^2-2\right)(\gamma_2+\gamma_3)^2 \\ &\quad - 2\gamma_2\gamma_3\Bigg(\sqrt{(1-4\delta_1)^2(\gamma_1(\gamma_2+\gamma_3)+\gamma_2\gamma_3)\left(\gamma_1\left(2-9\delta_1^2\right)^2(\gamma_2+\gamma_3)+\gamma_2\gamma_3(\delta_1(\delta_1+4)+2)^2\right)} \\ &\qquad - \gamma_2\gamma_3(4\delta_1-1)\left(9\delta_1^2-2\right)\Bigg) \\ &\quad + \gamma_1\Bigg(-2\gamma_2\sqrt{(1-4\delta_1)^2(\gamma_1(\gamma_2+\gamma_3)+\gamma_2\gamma_3)\left(\gamma_1\left(2-9\delta_1^2\right)^2(\gamma_2+\gamma_3)+\gamma_2\gamma_3(\delta_1(\delta_1+4)+2)^2\right)} \\ &\qquad - 2\gamma_3\sqrt{(1-4\delta_1)^2(\gamma_1(\gamma_2+\gamma_3)+\gamma_2\gamma_3)\left(\gamma_1\left(2-9\delta_1^2\right)^2(\gamma_2+\gamma_3)+\gamma_2\gamma_3(\delta_1(\delta_1+4)+2)^2\right)} \\ &\qquad + \gamma_2\gamma_3(4\delta_1-1)\left(\gamma_2\left(3\gamma_3(5\delta_1+2)^2+36\delta_1^2-8\right)+4\gamma_3\left(9\delta_1^2-2\right)\right)\Bigg)\Bigg), \end{aligned} \tag{53}$$

$$\alpha_2^* = \alpha_3^* = \frac{1}{3\gamma_1(4\delta_1-1)(5\delta_1+2)(\gamma_2+\gamma_3)(\gamma_1(\gamma_2(5\gamma_3\delta_1+2\gamma_3+2\delta_1)+2\gamma_3\delta_1)+2\gamma_2\gamma_3\delta_1)}$$
$$\Bigg(\gamma_1^2(4\delta_1-1)(\gamma_2+\gamma_3)\left(\gamma_2\left(3\gamma_3(5\delta_1+2)^2+\delta_1(\delta_1+4)+2\right)+\gamma_3(\delta_1(\delta_1+4)+2)\right)$$
$$+\gamma_1(\gamma_2+\gamma_3)\Bigg(2\gamma_2\gamma_3(4\delta_1-1)(\delta_1(\delta_1+4)+2)$$
$$-\sqrt{(1-4\delta_1)^2(\gamma_1(\gamma_2+\gamma_3)+\gamma_2\gamma_3)\left(\gamma_1\left(2-9\delta_1^2\right)^2(\gamma_2+\gamma_3)+\gamma_2\gamma_3(\delta_1(\delta_1+4)+2)^2\right)}\Bigg)$$
$$+\gamma_2\gamma_3\Bigg(\gamma_2\gamma_3(4\delta_1-1)(\delta_1(\delta_1+4)+2)$$
$$-\sqrt{(1-4\delta_1)^2(\gamma_1(\gamma_2+\gamma_3)+\gamma_2\gamma_3)\left(\gamma_1\left(2-9\delta_1^2\right)^2(\gamma_2+\gamma_3)+\gamma_2\gamma_3(\delta_1(\delta_1+4)+2)^2\right)}\Bigg)\Bigg), \tag{54}$$

$$\beta_1^* = \frac{1}{(\delta_1-1)(4\delta_1-1)(\gamma_2\gamma_3(\delta_1(7\delta_1+10)+4)-2\gamma_1(\delta_1(19\delta_1+4)-2)(\gamma_2+\gamma_3))}$$
$$\Bigg((\delta_1+2)\Bigg(-\sqrt{(1-4\delta_1)^2(\gamma_1(\gamma_2+\gamma_3)+\gamma_2\gamma_3)\left(\gamma_1\left(2-9\delta_1^2\right)^2(\gamma_2+\gamma_3)+\gamma_2\gamma_3(\delta_1(\delta_1+4)+2)^2\right)}\Bigg)$$
$$-\gamma_1\delta_1(4\delta_1-1)(\delta_1(9\delta_1-20)-10)(\gamma_2+\gamma_3)+\gamma_2\gamma_3\delta_1(2\delta_1+1)(3\delta_1+4)(4\delta_1-1)\Bigg), \tag{55}$$

$$\beta_2^* = \beta_3^* = \frac{1}{(\delta_1-1)(\gamma_2\gamma_3(\delta_1(7\delta_1+10)+4)-2\gamma_1(\delta_1(19\delta_1+4)-2)(\gamma_2+\gamma_3))}$$
$$\Bigg(-\sqrt{(1-4\delta_1)^2(\gamma_1(\gamma_2+\gamma_3)+\gamma_2\gamma_3)\left(\gamma_1\left(2-9\delta_1^2\right)^2(\gamma_2+\gamma_3)+\gamma_2\gamma_3(\delta_1(\delta_1+4)+2)^2\right)} \tag{56}$$
$$+\gamma_1(2\delta_1+1)(\delta_1(\delta_1+8)-2)(\gamma_2+\gamma_3)+\gamma_2\gamma_3(\delta_1(\delta_1(17\delta_1+8)-2)-2)\Bigg).$$

For the particular values $\gamma_1 = 3/4$, $\gamma_2 = 1/2$, $\gamma_3 = 1/3$, $\delta_1 = 5/4$ this yields, for instance,

$$\alpha_1^* = 0.408,\ \alpha_2^* = \alpha_3^* = 0.304,\ \beta_1^* = 0.812,\ \beta_2^* = \beta_3^* = 0.922.$$

The usual graphs confirm the existence of this asymmetric equilibrium.

D.2 Example of asymmetric *relative* reductive efforts per-unit-GDP

This example illustrates the use of an asymmetric contest based on GDP-normalised efforts as in M:(14). In our standard square-root/quadratic example environment, efficient efforts (e^*, f^*) are independent of the contest technology and thus given by the asymmetric but otherwise unchanged three-players version of M:(23) as[17]

$$
\begin{array}{l}
(\boldsymbol{e}^*, \boldsymbol{f}^*) \in \arg \underset{(\boldsymbol{e},\boldsymbol{f})}{max}\ u(\boldsymbol{e},\boldsymbol{f}) = \\
y(e_1) + y(e_2) + y(e_3) - \\
(e_1 + e_2 + e_3 - f_1 - f_2 - f_3)^2 - \\
\gamma_1 \left(e_1^2 + f_1^2\right) - \gamma_2 \left(e_2^2 + f_2^2\right) - \gamma_3 \left(e_3^2 + f_3^2\right)
\end{array}
\Leftrightarrow
\left\{
\begin{array}{l}
e_1^* = 0.513, f_1^* = 0.535, \\
e_2^* = 0.359, f_2^* = 0.267, \\
e_3^* = 0.288, f_3^* = 0.178.
\end{array}
\right.
\tag{57}
$$

Again for simplicity, we choose linear cost asymmetries $\gamma_3 = 1$, $\gamma_2 = 2/3$, $\gamma_1 = 1/3$ to differentiate players but asymmetric productive capability could be modelled in exactly the same way. Finally, we use relative pollution shares of $s_1 = 4/12$, $s_2 = 3/12$, $s_3 = 5/12$. For simplicity, we only present numerical results in this section.

For the three-players asymmetric contest, the redistribution pool is $P = \sum_{i=1}^{3}(1-\alpha_i)(\sqrt{e_i})$. This setup results in the three players' objectives

$$
\begin{array}{rcl}
u_1(\boldsymbol{e},\boldsymbol{f}) & = & \alpha_1 y(e_1) + p_1^1(\boldsymbol{e},\boldsymbol{f})\beta_1 P + (1 - p_1^1(\boldsymbol{e},\boldsymbol{f})) \left(\frac{x_2^r}{x_2^r + x_3^r} \frac{1-\beta_2}{2} + \frac{x_3^r}{x_2^r + x_3^r} \frac{1-\beta_3}{2} \right) P - \\
& & s_1(e_1 + e_2 + e_3 - f_1 - f_2 - f_3)^2 - \gamma_1 \left(e_1^2 + f_1^2\right), \\
u_2(\boldsymbol{e},\boldsymbol{f}) & = & \alpha_2 y(e_2) + p_2^1(\boldsymbol{e},\boldsymbol{f})\beta_2 P + (1 - p_2^1(\boldsymbol{e},\boldsymbol{f})) \left(\frac{x_1^r}{x_1^r + x_3^r} \frac{1-\beta_1}{2} + \frac{x_3^r}{x_1^r + x_3^r} \frac{1-\beta_3}{2} \right) P - \\
& & s_2(e_1 + e_2 + e_3 - f_1 - f_2 - f_3)^2 - \gamma_2 \left(e_2^2 + f_2^2\right), \\
u_3(\boldsymbol{e},\boldsymbol{f}) & = & \alpha_3 y(e_3) + p_3^1(\boldsymbol{e},\boldsymbol{f})\beta_3 P + (1 - p_3^1(\boldsymbol{e},\boldsymbol{f})) \left(\frac{x_1^r}{x_1^r + x_2^r} \frac{1-\beta_1}{2} + \frac{x_2^r}{x_1^r + x_2^r} \frac{1-\beta_2}{2} \right) P - \\
& & s_3(e_1 + e_2 + e_3 - f_1 - f_2 - f_3)^2 - \gamma_3 \left(e_3^2 + f_3^2\right).
\end{array}
\tag{58}
$$

The corresponding six first-order conditions induce the set of efficient efforts (57) as individual global maxima for the following set of design parameters $\langle \alpha, \beta, r^* = 2 \rangle$[18]

$$
\begin{array}{lll}
\alpha_1^* = 0.610, & \alpha_2^* = 0.740, & \alpha_3^* = 0.810, \\
\beta_1^* = 0.767, & \beta_2^* = 0.518, & \beta_3^* = 0.460.
\end{array}
\tag{60}
$$

The below figure confirms that these identify an equilibrium for the game where a ratio of asymmetric strategic variables is chosen.

D.3 Agreement formation

Our argument in proposition 3 uses the maximal threat $C' = \langle \alpha' = 1, \beta'; p(\boldsymbol{f})' \rangle$ to show that free-riding can always be discouraged. This extreme case, however, renders the agreement wholly ineffective if a punishment becomes necessary. The purpose of the following example is to show that

[17] Since the two-players asymmetric example with $\beta_1 + (1 - \beta_2) = 1 \Leftrightarrow \beta_1 = \beta_2$ is still 'too symmetric' to illustrate the workings of the asymmetric mechanism we switch into a three players example in this section.

[18] In the setup of section 4.2, the efficiency inducing parameter set of the symmetric version of this 'normalised' contest based on $x_i = f_i / y(e_i)$ is given by

$$
\alpha^* = \frac{11}{15}, \ \beta^* = \frac{1}{2} + \frac{1}{4r}. \tag{59}
$$

This is more favourable to the participants than the corresponding parameters under the standard contest M:(27).

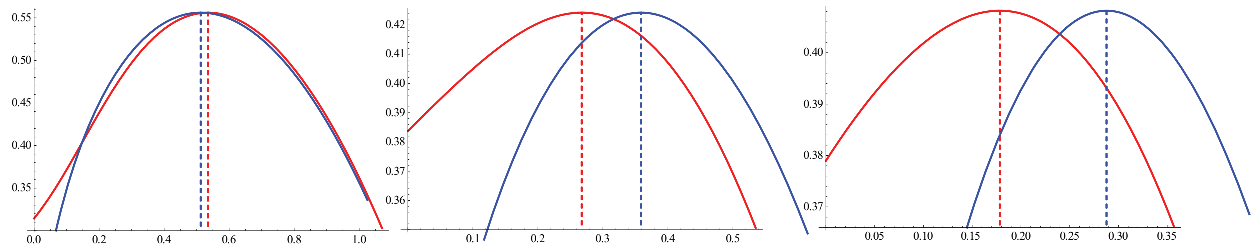

Figure 2: Efficient effort pairs as global maximisers of (58) for player 1's (left), player 2's (centre) and player 3's unilateral deviation utility (right). Red are abatement efforts, blue productive efforts. (The ranges shown contain all maxima.)

such severe measures are not generally needed. Typically, the punishment of a deserter leaves enough freedom to increase abatement levels over those realising under no agreement. As outlined in the previous sections, contract C implements efficient efforts. Consider now a deviation by some player which triggers $C' = \langle \alpha', \beta'; p(\boldsymbol{f})' \rangle$. Denote the equilibrium agreement utility attained by adhering to C' by $u'(\cdot)$ and the corresponding equilibrium efforts by $e'(\alpha', \beta', p')$, $f'(\alpha', \beta', p')$. By inflicting sufficient damage through $m(\cdot)$, we need to ascertain that free-riding utility $u_i^s(e_i^s, f_i^s)$—with the agreement members adhering to C'—is smaller than what participation in C gives, i.e.,

$$u_i^s(e_i^s, f_i^s|C') = y(e_i^s) - s_i m(e_i^s + (n-1)e'(\cdot) - f_i^s - (n-1)f'(\cdot)) - c_e(e_i^s) - c_f(f_i^s) \le u_i(e^*, f^*|C).$$

Since M:(23) implies that efforts e', f' are monotonic in α', β', payoff $u_i^s(\cdot|C')$ is continuous in α' and β'. Hence there exists an $\alpha' \in (\alpha, 1]$ which ensures the above inequality for suitable β' and $p'(\boldsymbol{f})$. Consider the case of $n+1$ players in our main example from section 5. Then, full participation efficient efforts are given by

$$e^* = \frac{n+2}{2 \times 2^{1/3}\left((2+n)(3+2n)^2\right)^{1/3}}, \quad f^* = \frac{n+1}{2 \times 2^{1/3}\left((2+n)(3+2n)^2\right)^{1/3}} \tag{61}$$

which are implemented by

$$\alpha^* = \frac{4\sqrt{e^*}(2e^* - f^*)(n+1) - 1}{n}, \quad \beta^* = \frac{1}{n+1} + \frac{2(e^* - 2f^*)f^*}{\sqrt{e^*}r(\alpha - 1)} \tag{62}$$

for the Tullock success function parameterised by r. This determines $u_i(e^*, f^*|C)$. For the deviation utility $u_i^s(e_i^s, f_i^s|C')$, the deviation efforts e^d, f^d are determined by the first-order conditions

$$2e^s = \frac{1}{2\sqrt{e^s}} + \frac{2(f^s + n(f'(\cdot) - e'(\cdot)) - e^s)}{n+1}, \quad f^s = \frac{e^s + n(e'(\cdot) - f'(\cdot))}{n+2} \tag{63}$$

in which $e'(\cdot), f'(\cdot)$ are the agreement equilibrium efforts in the agreement under C'. These functions $e'(\cdot), f'(\cdot)$, and therefore the damage they inflict on the deviator through $m(e^s + ne' - f^s - nf')$, are determined by

$$\alpha' = \frac{n\left(4\sqrt{e'}(e' + e^s - f^s + 2e'n - f'n) - 1\right) - 1}{n^2 - 1},$$

$$\beta' = \frac{\left(e^{2'}(4+8n) - 2f'(n-1)(f' + f^s + 2f'n - e^s) + 2e'(2e^s - 2f^s + f'(n-3)n) - \sqrt{e'}(n+1)\right)}{\left(4e^{2'}n(1+2n) - 4e'n(-e^s + f^s + fn) - \sqrt{e'}n(1+n)\right)}.$$

In our example setup, it turns out that participation is individually rational for any number of players. The details for the simplest case of three players (two in the agreement, one outside) are

$$e' = 0.302, f' = 0.195, e^s = 0.314, f^s = 0.132, (e^* = 0.273, f^* = 0.205)$$

for $C' = \langle \alpha' \approx 0.910, \beta' = 1; r' = 1\rangle$. If there is no agreement, efforts are $e^d = 0.303, f^d = 0.151$, so the 29% increased abatement efforts achieved by the punishment contract are substantial.

D.4 The efficient stochastic mechanism

This section illustrates the idea of variance compression in our usual two players example from section 5. Recall that under the efficient mechanism, player i expects the equilibrium utility from participating in the agreement[19]

$$u_i(\boldsymbol{e}^*, \boldsymbol{f}^*; \boldsymbol{\varepsilon}) = \alpha^* y_i(e^*, \varepsilon_i) + \frac{1}{2}P - \frac{1}{2}(2e^* - 2f^*)^2 - (e^*)^2 - (f^*)^2, \tag{64}$$

with $P = (1-\alpha^*)(y(e^*, \varepsilon_1) + y(e^*, \varepsilon_2))$. The main insurance argument rests on the simple observation that the variance of individual output σ^2 is higher than that of the pooled resource $\tilde{\sigma}^2$ to which all players contribute a share. The intuition is that the risk inherent in individual shocks with zero expectation evens out among several players. This is easiest to see if we assume i.i.d. shocks ε_i for which $\mathbb{V}\left[\sum_i \varepsilon_i\right] = \sum_i \mathbb{V}\left[\varepsilon_i\right]$, $i = 1, 2$. (In this environment it is safe to drop subscripts on ε.) The variance of the pooled output shocks is then given by (28) in the proof of proposition 5 as

$$\mathbb{V}_\varepsilon(\mathbb{E}(L|\varepsilon)) = \sigma^2 \frac{1 + (\alpha^*)^2(n-1)}{n} = \sigma^2 \frac{1 + (\alpha^*)^2}{2} < \sigma^2. \tag{65}$$

To this variance from the shocks alone we need to add the variance introduced through the contest prize structure. In expectation the contest does not play a role and, therefore, the expected contest payoff is just $\mathbb{E}[L|\varepsilon] = y(e^*, \varepsilon)$. Therefore, the contest variance is given by (30) as

$$\begin{aligned} \mathbb{V}(L|\varepsilon) = & \frac{1}{2}\left((1-\alpha)\beta^* 2y(e,\varepsilon) + \alpha^* y(e,\varepsilon) - \mathbb{E}[L|\varepsilon]\right)^2 \\ & + \frac{1}{2}\left((1-\alpha^*)(1-\beta^*)ny(e,\varepsilon) + \alpha^* y(e,\varepsilon) - \mathbb{E}[L|\varepsilon]\right)^2. \end{aligned} \tag{66}$$

From the law of total variance $\mathbb{V}\left[L\right] = \mathbb{E}\left[\mathbb{V}[L|\varepsilon]\right] + \mathbb{V}\left[\mathbb{E}[L|\varepsilon]\right]$, the condition (33) for an agreement member's payoff variance to be smaller than the standalone variance is in the two-players case

$$\sigma^2 \geq \mathbb{E}[y(e,\varepsilon)^2] \frac{2(1-\alpha^*)(1-2\beta^*)^2}{\alpha^* + 1}. \tag{67}$$

Substituting the equilibrium parameters of our standard square-root production and quadratic costs example (27) for $r = 2$ into the above gives the condition $\sigma^2 \geq 0.0039$. When this inequality is satisfied, the agreement equilibrium payoff has a smaller variance than standalone income.

[19] Notice that, since the efficient mechanism uses the ranking only for incentive purposes (based on expectations), the share $(1-\alpha)y(e_i, \varepsilon_i)$ that players commit to is stochastic.

References

Che, Y.-K. C., & Gale, I. (2000). Difference-form contests and the robustness of all-pay auctions. Games and Economic Behavior, 30, 22–43.

Engel, S. (2004). Achieving environmental goals in a world of trade and hidden action: the role of trade policies and eco-labeling. Journal of Environmental Economics and Management, 48(3), 1122–45.

Fang, K., Kotz, S., & Ng, K. (1987). Symmetric multivariate and related distributions. London: Chapman & Hall.

Grankvist, G., Dahistrand, U., & Biel, A. (2004). The impact of environmental labelling on consumer preference: Negative vs positive labels. Journal of Consumer Policy, 27(2), 213–30.

Harbaugh, R., Maxwell, J. W., & Roussillon, B. (2011). Label confusion: The groucho effect of uncertain standards. Management Science, 57, 1512–27.

Landsman, Z., & Valdez, E. (2003). Tail conditional expectations for elliptical distributions. North American Actuarial Journal, 7(4), 55–71.

Lerner, J., & Tirole, J. (2006). A model of forum shopping, with special reference to standard setting organizations. American Economic Review, 96, 1091–13.

Müller, A. (2001). Stochastic ordering of multivariate normal distributions. Annals of the Institute of Statistical Mathematics, 53(3), 567–75.

Rothschild, M., & Stiglitz, J. (1970). Increasing risk: I. a definition. Journal of Economic Theory, 2, 225–43.

Shaked, M., & Shanthikumar, J. (1994). Stochastic orders and their applications. Probability and Mathematical Statistics, San Diego: Academic Press.