Auctions with Signaling Bidders: Optimal Design and Information Disclosure*

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Abstract

We study optimal auctions in a symmetric private values setting, where bidders have signaling concerns: they care about winning the object and a receivers inference about their type. Signaling concerns arise in various economic situations such as takeover bidding, charity auctions, procurement and art auctions. We derive a decomposition of revenue into the standard revenue from the respective auction without signaling concern, and a signaling component. If two auctions have the same signaling value, they yield the same revenue. We then use information design to derive the optimal disclosure of bids. When signaling concerns are linear, it is optimal to reveal whether a bidder participated. Further disclosure of information is immaterial and every auction that reveals participation decisions yields the same highest revenue. With convex signaling concerns it is optimal to run a transparent auction which reveals all bids. With concave signaling concerns a trade-off arises between revealing as few information as possible about submitted bids and revealing participation decisions. The former increases the signaling value for the bidders, the latter allows for extracting a larger share of this signaling value from the bidders. Our methodology combines tools from mechanism design with tools from Bayesian persuasion.

Keywords: optimal auctions, revenue equivalence, Bayesian persuasion, information design

JEL classification: D44; D82

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1 Introduction

Since 1945, the *Hospices de Beaune*[1], in Burgundy (France), organizes an annual wine auction to raise money for local retirement houses and hospitals. In a special segment—the "pièce des Présidents"—a few barrels of wine are auctioned with the help of celebrities. Naturally, this segment draws considerable attention by the media, with extended press coverage. In the 2017 "pièce des Présidents" auction two barrels of *Corton Clos du Roi Grand Cru* were sold at a total price of €410,000. During the regular auction, the same wine realized prices ranging from €30,000 to €40,000 per barrel. Roughly speaking, public attention increased the price per barrel by 500%.[2] In this manuscript we propose signaling concerns of the bidders as an explanation for this pattern. Media coverage gives rise to a large audience, and the bidders care about their image held by this audience. In equilibrium, higher bids create a better image such that equilibrium bidding exhibits substantially larger bids when bidders have signaling concerns as compared to a situation without signaling concerns.

The wine auction introduced above is but one example of an auction where bidders have signaling concerns, i.e., bidders care about the object at sale but also about how they are perceived by others. Signaling incentives arise for several reasons in takeover bidding. First, competing firms issue equity or debt and the financial market draws inference from bidding behavior in the auction about the bidders' financial strength (Liu, 2012). Second, the bidding firms' managers have career prospects. Their future compensation is (partly) influenced by the inference potential employers draw from their performance in the auction (Giovannoni and Makris, 2014). Signaling concerns are also present in a procurement context: bidders want to be qualified for future tenders (Wan and Beil, 2009, Wan et al., 2012) and the organizers of these future tenders take the bidders' performance in past tenders as a signal of their intrinsic quality. Bidding in license or spectrum auctions is carefully observed by the stock market. The auction outcome is highly relevant for later competition in the market and in itself for the financial well-being of the bidding companies.[3] Finally, Mandel (2009) identifies signaling as an important aspect for buying and investing in artwork.

[1] https://www.beaune-tourism.com/discover/hospices-de-beaune-wine-auction

[2] Similar patterns arose in the previous years. Data for 2016 and 2017 are available at http://hospices-de-beaune.com/index.php?/hospicesdebeaune/content/download/3869/14085/version/1/file/catalogue_resultats_2016.pdf and http://hospices-de-beaune.com/index.php?/hospicesdebeaune/content/download/4248/15476/version/1/file/Vente+des+vins+-+Catalogue+des+r%C3%A9sultats+2017.pdf

[3] The 2019 German auction 5G spectrum was one of the most competitive among European countries, and raised €6.5 billions. Drillisch Netz AG became the new fourth mobile operator with two bids of €1.07 billion together for 70 MHz of spectrum. In the meantime its share price increased by 11%. See https://www.reuters.com/article/us-germany-telecoms/germany-raises-6-55-billion-euros-in-epic-5g-spectrum-auction-idINKCN1TD27D?edition-redirect=in and https://www.bundesnetzagentur.de/SharedDocs/Pressemitteilungen/EN/2019/20190612_spectrumauctionends.html

In this manuscript we study auction and information design when bidders care about the inference outsiders draw about their type. The auctioneer has two design tools to capitalize on the bidders' signaling concern: the payment rule and the disclosure rule. Bidders submit bids and the highest bid wins the object. The payment rule specifies each bidders' payment as a function of the submitted bids, e.g., first (or second) price auction where only the winner makes a payment, and this payment corresponds to the highest (or second highest) bid. Note that all bidders have signaling concerns, hence it is initially not clear whether a payment rule in which only the winner makes a payment can be optimal in our context. After the auction, it becomes public which bidder won the object. In addition, the auctioneer publicly discloses information about the submitted bids.[4] Such disclosure can range from no disclosure, where no further information is revealed, to fully disclosing all submitted bids alongside the bidders' identities. We illustrate some aspects of information disclosure with some practical examples.

In some environments laws govern the disclosure of information about submitted bids. Regulations for takeover bidding require public bids, that is every submitted bid has to be revealed to the public.[5] In contrast, in private procurement all details of the bidding process are treated as a trade secret. Merely the identity of the winning bidder becomes public, no information on bids and payments is revealed to the public. These examples capture the two extremes of information disclosure: full disclosure (takeover bidding) and no disclosure (private procurement). But information disclosure can also take forms inbetween these extremes. In public procurement regulations call for concealing individual bids, but the final price has to be published.[6] Because the final price is itself a function of submitted bids, there is some implicit information disclosure which depends on the selected payment rule. In a first-price auction with public revelation of the winner's payment, outsiders infer the winner's bid from her payment, and in case the equilibrium is strictly monotone thereby the winner's type. At the same time there is only noisy inference on losers' bids and types. Similarly, in a second-price auction with revelation of the winner's payment, outsiders observe the highest losing bid. This renders inference on *all* bidders noisy, because the identity of the highest losing bidder remains unknown. An all-pay auction provides an example of an auction format where information revelation is implicit (via prices) but still corresponds to full information disclosure as in the case of takeover bidding mentioned earlier.

[4]Formally, the auctioneer designs a *disclosure rule*, which maps a vector bids into publicly signals. See the model section for a formal definition and further examples.

[5]See Article 6 of Directive 2004/25/EC of the European Parliament and of the Council of 21 April 2004 on takeover bids: https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32004L0025&from=EN (last accessed March 3rd 2021).

[6]See Articles 21 and 22 as well as Annex V Part D of Directive 2014/24/EU of the European Parliament and of the Council of 26 February 2014 on public procurement: https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:02014L0024-20200101&from=EN (last accessed June 1st 2021).

In their seminal contributions Myerson (1981) and Riley and Samuelson (1981) show that—absent signaling concerns—every standard auction yields the same revenue. This is not necessarily the case when bidders care for signaling. Giovannoni and Makris (2014), Bos and Truyts (2021) study environments where different auction formats and bid disclosure policies yield different auction revenue, while for instance Goeree (2003), Molnar and Virag (2008), Katzman and Rhodes-Kropf (2008) find revenue equivalence in their respective settings. In this manuscript we study the simultaneous design of the auction (i.e., the payment rule) and information disclosure. We ask which information disclosure is optimal, and how the two design tools interact, i.e., for which disclosure rules does revenue equivalence obtain.

Our first finding is a decomposition of the auctioneer's revenue into the standard revenue from the auction without signaling concern, and a signaling component. The latter consisting of the bidders' total signaling value, minus the expected signaling value of a non-participating bidder, representing an endogenous outside option. This decomposition is a crucial step towards separating auction design from information design. The first component of revenue—the standard revenue—neither depends on the exact payment rule nor the disclosure rule. The remaining analysis then focuses on the signaling component of revenue. As long as the auctioneer applies exactly the same disclosure policy, revenue equivalence obtains. Importantly, this result holds upon directly disclosing information in bids. When in addition payments are public, as in the case of public procurement outlined above, different auction formats imply different information revelation and therefore differing revenue. From a practical perspective it is thus crucial to consider all channels of information disclosure, both direct via revealing information on bids, and indirect, via revealing information such as payments which indirectly link to bids.

We then continue studying optimal information disclosure. The signaling value depends on a specific *curvature* of bidders' signaling concern. We assume three relevant shapes in practice: linear, convex and concave. When signaling concerns are linear we show that any information disclosure which reveals whether a bidder participated yields the same revenue. In particular, it does not matter which additional information the auctioneer reveals about participating bidders. Under linearity the signaling value is independent of the information design, hence only the endogenous outside option yields differences in revenue. This outside option is minimal upon revealing whether a bidder participated. Via the payment rule the auctioneer then extracts *all* signaling value from bidders, independent of whether they end up winning the object. In practice, this can be achieved by charging an entry fee and publicly revealing all payments. Importantly, an auction format where only the winner makes a payment is not optimal, because also losers have a signaling concern and are willing to make payment which separates them from non-participants.

Matters are different when the signaling concerns is not linear. When bidders' preferences are convex, revenue increases in the amount of information the auction reveals. Hence, disclosing all bids is the optimal disclosure rule. As mentioned earlier, the all-pay auction is indirect way of achieving this, because disclosing the payments already reveals all relevant information. When signaling concerns are concave, deriving an optimal disclosure rule is less straightforward. The information revealed during the auction affects revenue in two ways. Revealing fewer information about bids increases the signaling value, which in turn increases revenue. But at the same time, the outside option becomes more attractive: if few information about submitted bids gets revealed the signaling value of a non-participating bidders gets more attractive. But with a more attractive outside option the auctioneer can extract only a smaller share of the signaling value from the bidders. The auctioneer can reduce the outside option by revealing whether a bidder participated (which is optimal for linear signaling concerns, see above) but this of course reduces the entire signaling value. A trade-off arises. We show that if participation is already fully observable it is optimal to reveal no additional information about submitted bids. Moreover, if participation in the auction is high enough (i.e., almost all bidder types participate in the auction), charging an entry fee is optimal: the reduction of the signaling value due to additional information revelation is small compared to the substantial reduction of the bidders' outside option.

On a technical level we adapt methods from Bayesian persuasion to work with distributions over posterior beliefs. Because ex-post beliefs play a prominent role in our analysis, we cannot directly apply standard tools from auction theory (e.g., Myerson, 1981), that use interim payments. Moving from beliefs to distributions over posterior beliefs allows to transfer the entire analysis to the interim stage and to combine standard tools from auction design with new tools from information design. This intermediate step allows us to move the entire analysis to the interim stage.

Auctions with signaling concerns have been recently investigated by Giovannoni and Makris (2014) and Bos and Truyts (2021).[7] The former consider auctions that reveal the winner's identity together with four disclosure policies: no information, the highest bid, the second highest bid, or all bids (each together with the respective bidder's identity). In particular, only a (strict) subset of bidders' payments is observable. Bos and Truyts compare second-price and English auctions, that reveal the winner's identity and her payment. Both studies compare specific auction formats and disclosure policies, and establish a failure of revenue equivalence. Our analysis, which covers all standard auctions, provides conditions under which revenue equivalence is restored via the use of an entry fee and optimal bid disclosure.

Our paper is also related to the literature on mechanism design with aftermarkets.

[7] There are also contributions about information transmission comparing specific auction formats followed by oligopoly competition. See, e.g., Goeree (2003), Das Varma (2003), Katzman and Rhodes-Kropf (2008) and von Scarpatetti and Wasser (2010).

Calzolari and Pavan (2006*a*,*b*) study contracting environments where the agent participates in an aftermarket. They find conditions under which no information release to the aftermarket is optimal. Dworczak (2020) analyzes an auction environment with a very general aftermarket. He restricts the analysis to cut-off mechanisms in which the information revealed about the winner only depends on the losers' bids. These mechanisms rule out disclosure of information contained only in the winner's bid, such as the price in a first-price auction or the entire vector of payments in the all-pay auction, which we show is optimal in some cases. Also Molnar and Virag (2008) study auctions with an aftermarket, to which only the winner is concerned by signaling (and there is no aftermarket if there is no winner). They show that it is optimal to reveal (conceal) the winner's type when the signaling incentive is convex (concave). In our setting all bidders care about how they are perceived, irrespective of whether they win. This brings about a novel decomposition of revenue into the standard non-signaling component and a signaling component, which can be analyzed using methods from information design. In addition, a new trade off arises from the bidders' endogenous outside option, implicitly given by the signaling value from abstention, which yields new insights in the concave and convex cases.

Information disclosure in auctions has first been analyzed in the setting of affiliated values by Milgrom and Weber (1982). Mechanism design problems with allocative and informational externalities have also been studied by Jehiel and Moldovanu (2000, 2001). The underlying assumption in this strand of literature is that an agent's valuation depends also on other agents' private information (and allocation). In our setting a bidder's utility is affected by the aftermarket's belief about her own valuation, while such beliefs have no impact in the literature on mechanism design with interdependent valuations.

The paper is organized as follows. Section 2 introduces the formal setting. In Section 3 we propose a decomposition of the seller's revenue to identify the play of *non-signaling* and *signaling* components. This preliminary result defines the outline of our analysis. Section 4 studies the case of linear signaling concerns. In Section 5 we derive optimal auctions when the signaling concerns are convex, and in Section 6 analyzes the concave case. We conclude and discuss our results in Section 7.

2 Formal Setting

We consider n bidders, who bid for a single object in an auction, and also care about the inference of an outside observer about their type.

Bidder i's valuation for the object (her 'type'), is denoted V_i, and is assumed i.i.d. and drawn according to a distribution function F with support on $[\underline{v}, \overline{v}] \subset \mathbb{R}_+$. Let $f \equiv F'$ denote the density function, $G \equiv F^{n-1}$ the distribution function of the highest order statistic among $n-1$ remaining valuations and $g \equiv G'$ the corresponding density function. Bidder i's realization of V_i, denoted v_i, is her private information, but the number of bidders and the distribution F are common knowledge.

We consider *standard auctions* in which each bidder submits a (non-negative) bid b_i, the highest bidder wins (ties broken at random), and bidder i's payment p_i depends on the entire vector of bids, i.e., $p_i(b_1, \ldots, b_n)$. We allow bidders to abstain from participation, which we formally model as 'bidding' $b_i = \emptyset$. Hence, the bid space is $B := \mathbb{R}_+ \cup \{\emptyset\}$. Note that a bidder who abstains does not make a payment and never wins the object (also if all other bidders abstain as well), while a bidder who bids $b = 0$ may end up winning the object and making a non-zero payment.

After the auction was conducted, the winning bidder's identity is publicly revealed.[8] In addition, the auctioneer chooses an *information disclosure policy* revealing information about the submitted bids. Formally, an information disclosure policy (S, σ) consists of a signal space S and a mapping $\sigma : B^n \to \Delta S$. Upon submitting bids $b_1, \ldots, b_n$ a signal $s \in S$ is drawn (potentially at random) from the distribution $\sigma(b_1, \ldots, b_n) \in \Delta S$. This formulation encompasses as extremes (i) a full revelation policy, where $S = B^n$ and $\sigma(b_1, \ldots, b_n) = \mathbb{I}_{(b_1,\ldots,b_n)}$, and (ii) a no revelation policy, where, e.g., $S = \{s\}$ is a singleton.[9] Many real-world examples of auctions do not feature explicit bid revelation, but implicit revelation via observable prices. For example, guidelines on public procurement demand that the final price is public.[10] Our model captures this as follows: for any standard auction let $S = \mathbb{R}^n_+$, and $\sigma(b_1, \ldots, b_n) = \mathbb{I}_{\left\{(p_1(b_1,\ldots,b_n),\ldots,p_n(b_1,\ldots,b_n))\right\}}$. That is, the auctioneer reveals each bidders' payment, and thus implicitly some information about submitted bids.[11] We denote $\mathcal{O} := (i^\star, s)$ the public outcome of the auction, where $i^\star$ is the winner's identity and s the signal revealed by the auctioneer.

Each bidder cares about winning the object, and about the inference of an outside observer, the 'receiver', about her type. This receiver can represent, e.g., the general public or press, business contacts or acquaintances of the bidder, or experts related to the object at sale. The receiver observes the auction outcome $\mathcal{O}$, but no further information about the bidding process, and forms a posterior belief about each bidder's type, denoted as $\mu_i(\mathcal{O})$. We assume that a bidder's utility depends on the receiver's belief only

[8]This is a natural assumption in most contexts. Only in some art auctions is it common to conceal the winner's identity.

[9]Technically there are many alternative variants for specifying these policies. E.g., no revelation can be obtained via an arbitrary signal space and a degenerate distribution σ which reveals the same signal, irrespective of the actual bids.

[10]See also Footnote 6.

[11]For instance, in a first-price auction the revelation of the winner's payment essentially reveals the winner's bid. In a second-price auction, the revelation of the winner's payment reveals that (i) the winner's bid was weakly higher, (ii) some losing bidder placed exactly this bid, (iii) all losing bidders placed a weakly lower bid. In an all-pay auction revelation of all payments is equivalent to revelation of all bids.

through the posterior mean, i.e., the expected value given the posterior distribution.[12] This is a reasonable assumption for instance in the context of takeover bidding, where a bidder's utility is affected by the decision of a competitive aftermarket of risk-neutral investors/firms (e.g., Liu, 2012, Giovannoni and Makris, 2014).[13] Formally, there is an increasing function $\Phi : [\underline{v}, \overline{v}] \to \mathbb{R}_+$, such that the bidder's utility is given by

$$u_i(v_i, \mathcal{O}) = \begin{cases} v_i - p_i + \Phi\left(\mathbb{E}\left(V_i|\mathcal{O}\right)\right), & \text{if } i = i^\star, \\ -p_i + \Phi\left(\mathbb{E}\left(V_i|\mathcal{O}\right)\right), & \text{if } i \neq i^\star. \end{cases}$$

The function Φ represents a reduced form of a (continuation) game in which the receiver chooses an action that directly affects the bidder's payoff. Note that a bidder's utility is not affected by the receiver's belief about other bidders' types. For instance, from an individual bidder's perspective it is equivalent to have either a different or the same receiver for each bidder.

Any standard auction defines a signaling game among bidders and the receiver. We consider symmetric perfect Bayesian equilibrium, consisting of the bidders' bidding strategies $\beta : [\underline{v}, \overline{v}] \to B$ and the receiver's belief $(\mu_1, \ldots, \mu_n)$. Each bidder's bidding strategy is optimal, given the other bidders' bidding and the receiver's beliefs. Also, the receiver's beliefs are Bayesian consistent with the bidding strategy. In our analysis we focus on equilibria in which (participating) bidders use strictly increasing bidding functions. This assumption is in line with the usual focus in auction theory. It thus allows for a straightforward comparison with the no-signaling benchmark. In the respective sections we briefly discuss whether this focus is restrictive. In the language of signaling games, we thus focus on separating equilibria, and do not emphasize the multiplicity of equilibria, selection and refinements. As a byproduct our analysis paves the way for proving existence of such equilibria for specific auction formats.

We conclude this section with an illustrative example for the bidders' utility functions. Suppose the bidder's valuation v is uniformly distributed on $[0, 1]$. The receiver cares about the bidder's characteristic θ, given by $\theta = \alpha v + (1 - \alpha)w$, where w is uniformly distributed on $[0, 1]$, independent of v, and $\alpha \in (0, 1)$. The receiver takes an action a to maximize her utility $U_R(a, \theta) = -(a - \theta)^2$. Given the auction outcome $\mathcal{O}$, the receiver chooses $a = \mathbb{E}(\theta|\mathcal{O})$. We have that

$$a = \mathbb{E}(\theta|\mathcal{O}) = \mathbb{E}(\alpha v + (1 - \alpha)w|\mathcal{O}) = \alpha\mathbb{E}(V|\mathcal{O}) + \frac{1 - \alpha}{2}$$

[12] Note that we do not assume that a bidder's type v_i *directly* affects the receiver's payoff. The receiver cares about some other characteristic of the bidder, which is correlated with the bidder's type. See also the example at the end of this section.

[13] Similar assumptions are made the context of Bayesian Persuasion (e.g., Dworczak and Martini, 2019, Rayo and Segal, 2010). An interpretation given there is that a single receiver takes an action, and the receiver preferences are, e.g., symmetric and single peaked, such that the decision equals the posterior mean.

If $\Psi(a)$ describes the impact of the receiver's action on the bidder's utility, we can define $\Phi\big(\mathbb{E}(V|\mathcal{O})\big) = \Psi\big(\alpha\mathbb{E}(V|\mathcal{O}) + \frac{1-\alpha}{2}\big)$. The shape of Ψ determines the shape of Φ. In the context of our introductory example of the *Hospices de Beaune*, θ may represent a bidder's altruism. Our model assumes that a bidder's altruism is correlated with her valuation for the wine auctioned, where α measures the degree of correlation.

3 Payoff-(non)-equivalence

The aim of this section is to establish a decomposition of the the seller's revenue. We show that the revenue is the sum of a *non-signaling* component, which corresponds to revenue obtained with the same auction format but with bidders who have no signaling concerns, and a *signaling* component. Such a decomposition implies that all differences in auction revenue stem from the second component, because the first component—the *no signaling* revenue—is the same across all auction formats.

In the following we expand standard arguments from auction theory to our setting with signaling bidders (Riley and Samuelson, 1981, Krishna, 2009). Fix some auction format A and suppose there is an equilibrium in which (i) bidder i participates if and only if $v_i \geq \tau$, and (ii) (participating) bidders follow the strictly increasing bidding strategy $\beta^A : [\tau, \overline{v}] \to \mathbb{R}$. Denote $m^A(v)$ the expected payment and $\mathcal{W}_\tau^A(v)$ the expected value from signaling of a bidder with valuation $v \geq \tau$. Non-participating bidders, i.e., bidders with $v < \tau$, have $m^A(v) \equiv 0$ and $\mathcal{W}_\tau^A(v) \equiv \mathcal{W}_{\tau,\emptyset}^A$. The value $\mathcal{W}_{\tau,\emptyset}^A$ denotes the (interim) expected signaling value of a bidder who abstains from participation.

Consider a bidder with valuation $v \geq \tau$. His expected payoff from mimicking type $\widetilde{v} \geq \tau$ is

$$\Pi(v, \widetilde{v}) = G(\tilde{v})v - m^A(\tilde{v}) + \mathcal{W}_\tau^A(\tilde{v}). \tag{1}$$

At equilibrium the bidder's payoff is $\Pi(v) := \Pi(v, v)$ and using the envelope theorem[14] it follows that

$$G(v)v - m^A(v) + \mathcal{W}_\tau^A(v) = \Pi(v) = \Pi(\tau) + \int_\tau^v G(x)\,\mathrm{d}x. \tag{2}$$

A bidder with valuation τ is indifferent whether to participate, if

$$\Pi(\tau) = \mathcal{W}_{\tau,\emptyset}^A. \tag{3}$$

Using (2) and (3) we express the *interim* expected payment of a bidder as follows

$$m^A(v) = G(v)v - \int_\tau^v G(x)\,\mathrm{d}x + \mathcal{W}_\tau^A(v) - \mathcal{W}_{\tau,\emptyset}^A = G(\tau)\tau + \int_\tau^v g(x)x\,\mathrm{d}x + \mathcal{W}_\tau^A(v) - \mathcal{W}_{\tau,\emptyset}^A. \tag{4}$$

[14] See Milgrom and Segal (2002). The objective in (1) is differentiable in v, and its derivative $G(\widetilde{v})$ is uniformly bounded.

Note that this expected payment depends on the auction format only via the signaling component $\mathcal{W}^A_\tau(v) - \mathcal{W}^A_{\tau,\emptyset}$. The auctioneer's revenue is $n\int_{\underline{v}}^{\overline{v}} m^A(v)\mathrm{d}F(v) = n\int_{\tau}^{\overline{v}} m^A(v)\mathrm{d}F(v)$. Define

$$Rev^{\mathcal{M}}(\tau) := n\int_{\tau}^{\overline{v}} \left(G(\tau)\tau + \int_{\tau}^{v} g(x)x\mathrm{d}x\right)\mathrm{d}F(v),$$

the revenue in the auction without signaling concern (see, e.g., Riley and Samuelson (1981)). Regarding the second term in 4, capturing the bidders' signaling values, recall that bidders preferences only depend on posterior means. Denote by H^A_τ the distribution over posterior means, that is induced by the equilibrium of the auction.[15] In particular, we have that $\int_{\underline{v}}^{\overline{v}} v\mathrm{d}H^A_\tau(v) = \int_{\underline{v}}^{\overline{v}} v\mathrm{d}F(v)$. As in the literature on Bayesian persuasion (e.g., Dworczak and Martini, 2019), we can rewrite a bidder's signaling value using the distribution over posterior means, instead of the interim values $\mathcal{W}^A_\tau$. Formally, we have that

$$\begin{aligned}\int_{\tau}^{\overline{v}} \left(\mathcal{W}^A_\tau(v) - \mathcal{W}^A_{\tau,\emptyset}\right)\mathrm{d}F(v) =& F(\tau)\mathcal{W}^A_{\tau,\emptyset} + \int_{\tau}^{\overline{v}} \mathcal{W}^A_\tau(v)\mathrm{d}F(v) - \mathcal{W}^A_{\tau,\emptyset}\\ =& \int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H^A_\tau(v) - \mathcal{W}^A_{\tau,\emptyset}\end{aligned}$$

Combining these steps yields the following Proposition.

Proposition 1. *Consider a standard auction A, in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increasing bidding strategy. The revenue in this auction is given by*

$$Rev^{\mathcal{M}}(\tau) + n\left(\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H^A_\tau(v) - \mathcal{W}^A_{\tau,\emptyset}\right). \tag{5}$$

It is immediate from Proposition 1 that differences in auction revenue are solely due to the signaling component, i.e., due to differences in the distribution over posterior means H^A_τ and the signaling value to non-participating bidders $\mathcal{W}^A_{\tau,\emptyset}$. The auction extracts the bidders' signaling value, given by the term $\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H^A_\tau(v)$. But signaling creates an endogenous outside option, given by $\mathcal{W}^A_{\tau,\emptyset}$, which the auctioneer cannot extract. Auction design thus corresponds to maximizing the bidders' signaling value—effectively performing information design—while keeping the outside option low. In the following we explore the optimal auction design in the light of formula (5) for linear, convex and concave signaling concerns.

[15]The distribution H^A_τ depends on the information disclosure policy and on the participation threshold τ, because all types $v < \tau$ are lumped together affecting the resulting posterior mean.

4 Linear Signaling Concerns

In this section, we consider a linear inference $\Phi(v) = \lambda v$, with $\lambda > 0$ the strength of a bidder's signaling concerns. Previous results indicate that even with linear inference revenue equivalence may fail. However, it remains open whether this failure can be attributed to the auction format or to the information disclosure policy. We show in Lemma 1 below, that two standard auctions with respective information disclosure policies yield the same revenue whenever the signaling value for non-participating bidders coincide.

Lemma 1. *Consider a standard auction A, in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increasing bidding strategy. With linear signaling concern, the revenue in this auction is given by*

$$Rev^{\mathcal{M}}(\tau) + n\left(\lambda\mathbb{E}(V) - \mathcal{W}^{A}_{\tau,\emptyset}\right). \tag{6}$$

Proof. From Proposition 1 we have that the revenue equals (5). Furthermore, we have that

$$\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H^{A}_{\tau}(v) = \lambda\int_{\underline{v}}^{\overline{v}} v\mathrm{d}H^{A}_{\tau}(v) = \lambda\int_{\underline{v}}^{\overline{v}} v\mathrm{d}F(v) = \lambda\mathbb{E}(V),$$

where the middle equality uses the fact the induced distribution over posterior means preserves the mean of the original distribution F. □

With linear signaling concerns the bidders' signaling value does not depend on the auction format. As in the literature on Bayesian persuasion, any information structure is equally valuable for a risk-neutral sender. So why does auction revenue still differ when bidders signaling concern is linear? The bidders' outside option $\mathcal{W}^{A}_{\tau,\emptyset}$ still depends on the revealed information. Without further information, outside observes cannot distinguish non-participating bidders from participating losers, which inflates the value $\mathcal{W}^{A}_{\tau,\emptyset}$ and thereby reduces auction revenue. Note that the value $\mathcal{W}^{A}_{\tau,\emptyset}$ already differs between first and second-price auctions when these also reveal the winner's payment. In a first-price auction we infer that all losers have a type below the winner's, which is perfectly revealed from his payment. In a second-price auction the winner's payment reveals the second-highest type, and a loser has either exactly this type or a lower type.

Because revenue decreases in bidders' outside option $\mathcal{W}^{A}_{\tau,\emptyset}$, the auctioneer aims at revealing information about submitted bids such that this value reduces. By revealing whether a bidder submitted a bid the auctioneer makes participation perfectly observable. This allows for reducing the outside option to the minimum feasible, and therefore maximizes auction revenue.

Proposition 2 (Revenue Equivalence)**.** *Consider a standard auction in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increas-*

ing bidding strategy. With linear signaling concern and a disclosure policy that reveals whether a bidder participated the revenue equals

$$Rev^{\mathcal{M}}(\tau) + n\lambda\big(\mathbb{E}(V) - \mathbb{E}(V|V < \tau)\big). \tag{7}$$

Moreover, no other auction cum disclosure policy yields higher revenue.

Proof. Note that $\mathcal{W}^{A}_{\tau,\emptyset} \geq \lambda\mathbb{E}(V|V < \tau)$, because by assumption all types below τ do not participate, and are hence lumped together. Choosing $S = \{0,1\}^n$ and $\sigma_i(b_1,\dots,b_n) = 0$ if $b_i = \emptyset$ and equal to one otherwise. With such a disclosure policy, the inference about bidder i's type is $\mathbb{E}(V|V < \tau)$ if the signal reveals bidder i did not participate. Hence, we get equality in the above formula. Plugging this value into (6) yields (7). □

A commonly applied way for making participation observable is to charge an *entry fee* and reveal all bidders payments. This way, an outsider who observes is able to tell apart losing bidders from non-participants, because losing bidders did make a payment–the entry fee–while non participants did not make a payment at all. In particular it then does not matter which specific auction design is in place, i.e., which inference can be drawn from the winner's payment etc. It is already sufficient to reveal only which bidders paid the entry fee, further revelation of bids is not required, but also does not harm the seller. Note that setting a *reserve price* does not achieve maximal revenue, if the auctioneer does not make public which bids actually exceeded the reserve price.

Next we want to compare optimal levels of participation among situations with and without signaling concerns. We focus on auctions with optimal information disclosure, i.e., auctions which make participation decisions public, as these attain the maximal revenue determined in Proposition 2. Denote with $\tau^{\star}(\lambda)$ the optimal level of participation under signaling strength λ.[16] Note that $\tau^{\star}(0) = \tau^{\mathcal{M}}$, where $\tau^{\mathcal{M}}$ denotes the optimal participation cut-off in an auction without signaling concerns. The following Corollary shows that the optimal level of participation (weakly) increases in the signaling strength and there exists a finite threshold of signaling strength which makes full participation optimal.

Corollary 1 (Optimal Participation)**.** *Assume virtual valuations are increasing. We have that*

(*i*) $\tau^{\star}(\lambda) \leq \tau^{\star}(\lambda') < \tau^{\mathcal{M}}$ *for all* $\lambda > \lambda' > 0$.

(*ii*) *There exists* $\overline{\lambda}$ *such that* $\tau^{\star}(\lambda) = \underline{v}$, *for all* $\lambda > \overline{\lambda}$.

[16] In general there may not be a unique optimal level of participation. Our assumption of increasing virtual valuations in Corollary 1 guarantees both existence and uniqueness of $\tau^{\star}(\lambda)$.

Proof. From Riley and Samuelson (1981) we know that

$$\big(Rev^{\mathcal{M}}\big)'(\tau) = nF^{n-1}(\tau)\big(1 - F(\tau) - \tau f(\tau)\big) = -nf(\tau)F^{n-1}(\tau)\left(\tau - \frac{1 - F(\tau)}{f(\tau)}\right).$$

Provided that virtual valuations $v - \frac{1-F(v)}{f(v)}$ are strictly monotone we have that $Rev^{\mathcal{M}}(\tau)$ has a unique maximum $\tau^{\mathcal{M}}$. Furthermore, $\big(Rev^{\mathcal{M}}\big)'(\tau) < 0$ for all $\tau > \tau^{\mathcal{M}}$, and $\big(Rev^{\mathcal{M}}\big)'(\tau) > 0$ for all $\tau \in (\underline{v}, \tau^{\mathcal{M}})$. Together with the observation that $\mathbb{E}(V|V < \tau)$ strictly increases in τ, (i) follows. To prove (ii) note that the derivative of $Rev^{\mathcal{M}}(\tau)$ is bounded. Hence, as soon as λ becomes sufficiently large we have that $\lambda \frac{\partial}{\partial \tau}\mathbb{E}(V|V < \tau) > \big(Rev^{\mathcal{M}}\big)'(\tau)$ for all $\tau > \underline{v}$ and thus full participation maximizes revenue in the auction with signaling concerns. □

We conclude this section with an illustration of our results for the first- and the second-price auction with observable payments (i.e., where the disclosure policy is such that all payments get publicly revealed).

Example 1 (Equilibrium of the first-price auction)**.** Consider a first-price auction with entry fee φ.[17] The critical type τ pays the entry fee φ and places a zero bid. Equations (3) and (4) together with $\beta(\tau) = 0$ yields

$$\beta(v) = \int_{\tau}^{v} \frac{xg(x)}{G(v)}dx + \frac{\mathcal{W}_{\tau}(v) - \mathcal{W}_{\tau}(\tau)}{G(v)}, \qquad \forall v > \tau.$$

The signaling value $\mathcal{W}_{\tau}(v)$ can be expressed as follows

$$\begin{aligned}\mathcal{W}_{\tau}(v) =& \lambda G(v)v + (1 - G(v))\frac{\lambda}{1 - G(v)}\int_{v}^{\overline{v}} \mathbb{E}(V|\tau < V < x)dG(x) \\ =& \lambda G(v)v + \int_{v}^{\overline{v}}\int_{\tau}^{x} \frac{\lambda y}{F(x) - F(\tau)}dF(y)dG(x).\end{aligned}$$

Hence,

$$\beta(v) = \int_{\tau}^{v} \frac{xg(x)}{G(v)}dx + \lambda\frac{G(v)v - G(\tau)\tau}{G(v)} + \int_{\tau}^{v}\int_{\tau}^{x} \frac{\lambda y}{F(x) - F(\tau)}dF(y)dG(x).$$

Bos and Truyts (2019, Proposition 1) show that these bidding strategies indeed constitute an equilibrium of the first-price auction.

Example 2 (Non-existence of monotone equilibria in the second-price auction)**.** Consider a second-price auction with two bidders and valuations uniformly distributed on $[0, 1]$.

[17] The superscript A is omitted in Example 1 and 2, as they both focus on a specific auction format.

Using equations (3) and (4) we get that

$$\int_\tau^v \beta(x)dx = \int_\tau^v xdx + \mathcal{W}_\tau(v) - \mathcal{W}_\tau(\tau),$$

which implies $\beta(v) = v + \mathcal{W}_\tau'(v)$. In the second-price auction we have for $v \geq \tau$

$$\mathcal{W}_\tau(v) = \tau\lambda\mathbb{E}(V|V \geq \tau) + \int_\tau^v \lambda\mathbb{E}(V|V \geq x)dx + (1-v)\lambda v.$$

Hence,

$$\beta(v) = v + \mathcal{W}_\tau'(v) = v + \lambda\frac{3}{2}(1-v)$$

For $\lambda > 2/3$ the bidding strategy is *decreasing* in v, hence an *increasing* equilibrium does not exist. Consequently, a second-price auction fails to allocate the good efficiently and is not optimal. With two bidders participation is automatically observable in a second-price auction, because the losing bidders participation decision can be inferred from the price. Hence, the non-existence result does not depend on whether participation is observable.

5 Convex Signaling Concerns

In this section, we consider convex signaling concerns, i.e., the case where Φ is strictly increasing and convex. With convex signaling concern it is no longer true that the total signaling value is independent of the auction design. Yet, any auction induces a distribution over posterior means that averages to the same mean, hence is a mean-preserving spread. With a convex Φ it is then optimal to disclose as much information as possible, because this increases the signaling value. The maximal amount of information that an auction can disclose corresponds to fully disclosing the types of participating bidders, as no information from non-participating bidders is obtained.

Proposition 3 (Optimal auction under convexity). *Consider a standard auction, in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increasing bidding strategy. With convex signaling concern, the revenue in this auction is at most*

$$Rev^{\mathcal{M}}(\tau) + n\left(\int_\tau^{\overline{v}} \Phi(v)dF(v) - (1-F(\tau))\Phi(\mathbb{E}[V|V \leq \tau])\right). \quad (8)$$

Any standard auction with full revelation of all bids exhibits the described equilibrium and attains the revenue bound.

Proof. From Proposition 1 we have that revenue equals (5). Define $H_\tau^{\max}$ the distribution over posterior means as follows

$$H_{\tau}^{\max}(v)=\begin{cases}0, & \text{if } \underline{v}\leq v<\mathbb{E}[V|V\leq\tau],\\ F(\tau), & \text{if } \mathbb{E}[V|V\leq\tau]\leq v\leq\tau,\\ F(v), & \text{if } \tau<v\leq\overline{v}.\end{cases} \tag{9}$$

The distribution $H_{\tau}^{\max}$ has a mass point at $\mathbb{E}[V|V\leq\tau]$ and otherwise corresponds to the prior F for values above τ. $H_{\tau}^{\max}$ is the distribution over posterior beliefs induced by a rule that fully reveals the type of a participating bidder, and otherwise discloses no further information. Because the observer can never distinguish the types of nonparticipating bidders, we have that $H_{\tau}^{\max}$ is a mean-preserving spread of H_{τ}^{A}. Convexity of Φ thus implies

$$\int_{\underline{v}}^{\overline{v}}\Phi(v)dH_{\tau}^{A}(v)\leq\int_{\underline{v}}^{\overline{v}}\Phi(v)dH_{\tau}^{\max}(v)=F(\tau)\Phi(\mathbb{E}[V|V\leq\tau])+\int_{\tau}^{\overline{v}}\Phi(v)dF(v). \tag{10}$$

Together with our previous observation that $\mathcal{W}_{\tau,\emptyset}^{A}\geq\Phi(\mathbb{E}[V|V\leq\tau])$ the revenue bound (8) follows.

To show the revenue bound (8) can be attained, consider a standard auction with a full disclosure policy, i.e., where all bids are disclosed. Denote $\beta^{\mathcal{M}}(v)$ the respective equilibrium bid in the auction without signaling concern, and $m^{\mathcal{M}}(v)$ the respective expected payment upon bidding as type v, i.e., placing the bid $\beta^{\mathcal{M}}(v)$. Now define $\beta(v)=\beta^{\mathcal{M}}(v)+x$ such that for the expected payment in a separating equilibrium of the auction with signaling we have $m(v)=m^{\mathcal{M}}(v)+\Phi(v)$. By definition, β is strictly increasing. Hence, revealing the bid in fact reveals the type. It remains to show that bidding according to *beta* indeed forms an equilibrium. Specify the system of beliefs such that (i) a bidder who did not participate (i.e., bids $b=\emptyset$) has expected type $\mathbb{E}(V|V\leq\tau)$ (formally, the bidder's type is distributed according to the truncation of F on the interval $[\underline{v},\tau]$), (ii) upon observing bid b the receiver beliefs to face type $\beta^{-1}(v)$ (if such a type exists) and (iii) for all other bids the receiver believes the bidder's type is $\underline{v}$. Note that the expected payoff of type v who mimicks type v' is

$$G(v')v-m(v')+\Phi(v')=G(v')v-m^{\mathcal{M}}(v')$$

As $\beta^{\mathcal{M}}$ was an equilibrium of the auction without signaling, the above objective is maximized upon bidding bidding $\beta(v)$, i.e., bidding as the true type. We have thus verified that the bidding strategy β together with the specified system of beliefs forms an equilibrium. □

Part of our analysis of finding the optimal auction amounts to determining the optimal information structure. We maximize (5) over the set of distributions H_{τ}^{A} which are a mean-preserving spread of $H_\tau^{\max}$, where $H_\tau^{\max}$ is the distribution over posterior means of a disclosure policy that reveals the true type v if $v \geq \tau$ and lumps all other types on one signal. Put differently, $H_\tau^{\max}$ arises from disclosing the (maximal) information gathered by running the auction. Dworczak and Martini (2019) provide a solution to this problem, which in our case of a convex signaling function Φ yields full disclosure. But notice an important difference to their analysis: the signaling value of non-participating bidders $\mathcal{W}_{\tau,\emptyset}$ negatively enters the auctioneer's objective. In the convex case, maximizing $\int_{\underline{v}}^{ov} \Phi(v)\mathrm{d}H(v)$ coincidentally minimizes $\mathcal{W}_{\tau,\emptyset}^{A}$ and hence the solution obtains.

Proposition 3 implies revenue equivalence with the additional requirement that the auction is augmented by an optimal disclosure policy that reveals all submitted bids. This finding has a couple of implications relevant in practice. First, without an optimal disclosure policy revenue is typically lower than Equation (8). An example where an optimal disclosure policy is used is takeover bidding, where regulations require all bidding to be public. Our findings imply, that the specific payment rule has no further impact on revenue. Second, with a given non-optimal disclosure policy revenue equivalence does not obtain. Take for instance a first- and a second-price auction that reveal the winner's payment. Inference on winning and losing bidders' types is different, hence the signaling values differ. As a consequence, when the choice of a disclosure policy is restricted, auction design is critical. For instance, in public procurement regulations require disclosure of the final price but do not allow any further revelation of bids. When bidders have non-linear signaling concerns it is then a non-trivial task to decide on the optimal auction format.

As in the case of linear signaling concerns, we can look for the optimal participation threshold $\tau^\star$ in the auction. The bidders' signaling concerns induce the auctioneer to reduce the threshold for participation below the optimal level without signaling.

Corollary 2. *Assume virtual valuations are increasing. In the revenue maximizing auction more bidders participate than if bidders had no signaling concern, i.e.* $\tau^\star < \tau^{\mathcal{M}}$.

Proof. It follows similar steps as the proof of Corollary 1. □

Remark 1. Under convex (and linear) signaling it is also possible to show that auctions maximize the seller's revenue across all mechanisms. To see this, assume we were to analyze direct mechanisms that in addition disclose a signal for each bidder (as in Dworczak (2020)). To simplify, assume in addition that transfers are unobservable to the receiver. Similar to (2), we can prove a payoff-equivalence and decompose revenue in a non-signaling and a signaling component. The signaling component is maximal for full disclosure of participating types (or any disclosure plus observable participation in the linear case). Under the usual regularity condition, the non-signaling component is maximized by allocating the good to the bidder with the highest value above a reserve value. Note that full disclosure of participating types can be achieved, e.g., via disclosing a bidder's interim payment.

6 Concave Signaling Concerns

In this section, we consider concave signaling concerns. From Proposition 1 we have that the revenue can be decomposed as in (5). Information disclosure directly affects the second and third components, respectively $\int \Phi(v)\mathrm{d}H_\tau^A(v)$ and $\mathcal{W}_{\tau,\emptyset}^A$. Under concavity, the signaling value $\int \Phi(v)\mathrm{d}H_\tau^A(v)$ increases if the auctioneer discloses fewer information about bidders. At the same time, revealing less information *increases* the outside option, i.e., the signaling value of non-participating bidders $\mathcal{W}_{\tau,\emptyset}^A$.

More precisely, consider first the signaling value, given by the term $\int \Phi(v)\mathrm{d}H_\tau^A(v)$. Information disclosure amounts to choosing a distribution over posterior means H_τ^A, where H_τ^A is a mean-preserving spread of the *minimal information* distribution $H_\tau^{\min}$. The latter distribution is given by the policy that discloses only the winner's identity, if the auction has a winner. Because the inference function Φ is concave, the signaling value is maximal for $H_\tau^A = H_\tau^{\min}$. In words, the signaling value is maximal when the auction reveals no additional information beyond the winner's identity (which has to be revealed by assumption). Next consider the term $\mathcal{W}_{\tau,\emptyset}^A$, the expected inference of a bidder who does not participate. As before, $\mathcal{W}_{\tau,\emptyset}^A$ is minimal if the auction reveals whether a bidder participated. In contrast to the convex case, maximizing the signaling value and minimizing $\mathcal{W}_{\tau,\emptyset}^A$ conflict with each other. In particular, revealing whether a bidder participated in the auction reveals *more* information than only revealing the winner's identity. In general this yields a non-trivial trade-off without a straightforward solution.

Define H_τ^P the distribution over posterior means that arises from a disclosure policy which reveals the winner's identity and whether a bidder participated in an auction with participation threshold τ. Note that $H_{\underline{v}}^P = H_{\underline{v}}^{\min}$. Moreover, define

$$Rev^P(\tau) = Rev^{\mathcal{M}}(\tau) + n\Big(\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H_\tau^P(v) - \mathbb{E}[V|V<\tau]\Big). \tag{11}$$

Proposition 4. *Consider a standard auction, in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increasing bidding strategy.*

(i) If participation is fully observable we have that $Rev(\tau) \leq Rev^P(\tau)$.

(ii) There exists $\tau' > \underline{v}$ such that $Rev(\tau) \leq Rev^P(\tau)$ whenever $\tau < \tau'$.

Proof. From Proposition 1 we have that revenue equals (5). With observable participation we have $\mathcal{W}_{\tau,\emptyset}^A = \mathbb{E}[V|V<\tau]$, independent of the specific auction format. Furthermore, because Φ is concave the signaling value $\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H_\tau^A(v)$ is maximal when only the winner's identity is disclosed in addition, i.e., when $H_\tau^A = H_\tau^P$. This proves (i).

To prove (ii), note that for every auction in which participation is not fully observable we have $\mathcal{W}_{\tau,\emptyset}^A > \mathbb{E}[V|V<\tau]$. Moreover, for $\tau = \underline{v}$ we have $\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H_{\underline{v}}^A(v) \leq \int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H^{\min}(v)$ by concavity of Φ. Hence, $Rev(\underline{v}) < Rev^P(\underline{v})$. Both $\mathcal{W}_{\tau,\emptyset}^A$ and

$\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H_\tau^A(v)$ are continuous in the participation threshold τ, hence (ii) follows by continuity from the previous assertion.

□

Proposition 4 derives an upper bound for the revenue if either participation is fully observable, or many bidder types participate. In other cases, i.e., when few bidder types participate in the auction, other information disclosure policies than revealing the winner's identity and whether a bidder participated may lead to higher revenue. In particular this may require fewer information to be provided, i.e., concealing participation decisions.

Proposition 4 reveals a fundamental difference between pure information design and mechanism design with information disclosure. In information design the sender has costless access to all information, while in mechanism design the information is privately held by the agents. In our setting the auctioneer benefits from revealing additional information, namely whether a bidder participated. Such disclosure reduces the value of the bidders' outside option, i.e., the expected signaling value from non-participation. The reduced outside option allows the auctioneer to extract more revenue from bidders. The benefit from revealing the bidders' participation is larger, the lower the participation threshold τ. For $\tau \approx \underline{v}$ it becomes optimal to disclose only the winner's identity and all participation decisions.

The bound derived in Proposition 4 stems from a disclosure policy that reveals only the winner's identity, and a list of participating bidders. In particular, revealing additional information such as the winner's payment in first- or second-price auction reduces revenue. In some contexts it is mandatory to disclose such payments. Does this necessarily imply that revenue strictly reduces? We show that in theory there is a remedy to eliminate the adverse effects of information leakage via observable payments.

Proposition 5. *For every $\varepsilon > 0$ and every τ there is an auction with mandatory disclosure of all payments that exhibits an equilibrium with strictly increasing bidding strategies, for which $Rev(\tau) > Rev^P(\tau) - \varepsilon$.*

Proof. Consider the following variant of a first-price auction: Bidders submit non-negative bids, the bidder submitting the highest bid wins and with exogenous probability $1 - \varepsilon$ makes no payment, but pays his own bid with probability ε.[18] Every bidder has to pay the entry fee φ before submitting a bid. The expected profit of a bidder of type v upon entering the auction and bidding as if he was type v' is

$$\Pi(v|v') = G(v')(v - \varepsilon\beta(v')) + \mathcal{W}_\tau(v') - \varphi.$$

[18] The superscript A is omitted in the proof, as it goes through a specific first-price auction.

From the first-order condition we get

$$\beta^\star(v) = \frac{1}{\varepsilon}\beta^{\mathcal{M}}(v) + \frac{\mathcal{W}_\tau(v) - \mathcal{W}_\tau(\tau)}{\varepsilon G(v)},$$

where $\beta^{\mathcal{M}}$ is the bidding strategy in a first-price auction without signaling and entry fee that induces only types above τ to participate. Note that we have used the fact that $\beta(\tau) = 0$, which is true because in equilibrium type τ only wins the auction when no other bidder enters and is thus not willing to bid a strictly positive amount. Furthermore, to induce participation for all types above τ the fee has to satisfy

$$\mathcal{W}_{\tau,0} = G(\tau)\tau + \mathcal{W}_\tau(\tau) - \varphi \quad \Leftrightarrow \quad \varphi = G(\tau)\tau + \mathcal{W}_\tau(\tau) - \mathcal{W}_{\tau,\emptyset}.$$

The revenue is thus given by

$$\begin{aligned}
Rev^\varepsilon(\tau) &= n(1-F(\tau))\varphi + (1-F^n(\tau))\mathbb{E}\big[\varepsilon\beta^\star(V_1)|V_1 \geq \tau\big] \\
&= n(1-F(\tau))\big(G(\tau)\tau + \mathcal{W}_\tau(\tau) - \mathcal{W}_{\tau,\emptyset}\big) + \int_\tau^{\overline{v}} \left(\beta^{\mathcal{M}}(s) + \frac{\mathcal{W}_\tau(s) - \mathcal{W}_\tau(\tau)}{G(s)}\right)\mathrm{d}F^n(s) \\
&= Rev^{\mathcal{M}}(\tau) + n(1-F(\tau))(\mathcal{W}_\tau(\tau) - \mathcal{W}_{\tau,\emptyset}) + n\int_\tau^{\overline{v}} \big(\mathcal{W}_\tau(s) - \mathcal{W}_\tau(\tau)\big)f(s)\mathrm{d}s \\
&= Rev^{\mathcal{M}}(\tau) + n\left[F(\tau)\mathcal{W}_{\tau,\emptyset} + \int_\tau^{\overline{v}} \mathcal{W}_\tau(s)f(s)\mathrm{d}s - \mathcal{W}_{\tau,\emptyset}\right]. \qquad (12)
\end{aligned}$$

Note that

$$\begin{aligned}
\mathcal{W}_\tau(s) &= \sum_{k=1}^n \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{F_k(s|\tau)\cdot\Big(\varepsilon\Phi(s) + (1-\varepsilon)\Phi\big(v_{W,k}\big)\Big) \\
&\qquad + (1-F_k(s|\tau))\cdot\Big(\varepsilon\int_s^{\overline{v}}\Phi\big(v_{L,k}(x)\big)\mathrm{d}F_k(x|\tau) + (1-\varepsilon)\Phi\big(v_{L,k}\big)\Big)\Big\} \\
&= \sum_{k=1}^n \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{F_k(s|\tau)\Phi\big(v_{W,k}\big) + (1-F_k(s|\tau))\,\Phi\big(v_{L,k}\big)\Big\} \\
&\quad -\varepsilon\sum_{k=1}^n \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{F_k(s|\tau)\Big(\Phi(s) - \Phi\big(v_{W,k}\big)\Big) \\
&\qquad +(1-F_k(s|\tau))\Big(\int_s^{\overline{v}}\Phi\big(v_{L,k}(x)\big)\mathrm{d}F_k(x|\tau) - \Phi\big(v_{L,k}\big)\Big)\Big\},
\end{aligned}$$

where $\mathcal{B}_{n-1,F(\tau)}(k-1) := \binom{n-1}{k-1}F(\tau)^{n-k}(1-F(\tau))^{k-1}$, $v_{W,k} := \mathbb{E}[V_1|V_k \geq \tau > V_{k+1}]$, $v_{L,k} := \mathbb{E}[V|V_1 > V \geq V_k \geq \tau > V_{k+1}]$, $v_{L,k}(s) := \mathbb{E}[V|V_1 = s, s > V \geq V_k \geq \tau > V_{k+1}]$ and $F_k(v|\tau) := \big(\frac{F(v)-F(\tau)}{1-F(\tau)}\big)^{k-1}$ for all $k = 1, \ldots, n$ denotes the conditional probability of the maximum of the $k-1$ other bids if all of these exceed τ. Hence,

$$\begin{aligned}\int_\tau^{\overline{v}} \mathcal{W}_\tau(s)f(s)\mathrm{d}s &= \sum_{k=1}^{n}\mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{\tfrac{1}{k}\Phi\big(v_{W,k}\big)+\tfrac{k-1}{k}\Phi\big(v_{L,k}\big)\Big\} \\ &\quad -\varepsilon\sum_{k=1}^{n}\mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{\int_\tau^{\overline{v}} F_k(s|\tau)\Big(\Phi(s)-\Phi\big(v_{W,k}\big)\Big)f(s)\mathrm{d}s \\ &\quad +\int_\tau^{\overline{v}}(1-F_k(s|\tau))\Big(\int_s^{\overline{v}}\Phi\big(v_{L,k}(x)\big)\mathrm{d}F_k(x|\tau)-\Phi\big(v_{L,k}\big)\Big)f(s)\mathrm{d}s\Big\} \\ &= \sum_{k=1}^{n}\mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{\tfrac{1}{k}\Phi\big(v_{W,k}\big)+\tfrac{k-1}{k}\Phi\big(v_{L,k}\big)\Big\}-\varepsilon C,\end{aligned}$$

where by concavity of Φ and compactness of the support of the bidders' valuations we have $C>0$ and finite. Plugging the above expression back into (12) and noting that $\mathcal{W}_{\tau,\emptyset}=\mathbb{E}[V|V<\tau]$ (because participation is observable) yields $Rev^{\varepsilon}(\tau)=Rev^{P}(\tau)-\varepsilon C\to Rev^{P}(\tau)$ as $\varepsilon\to 0$. □

Remark 2. With concave signaling concerns an auction does not necessarily constitute an optimal mechanism, hence our focus on equilibria in strictly increasing strategies is restrictive. To see this let us consider the following example with 2 bidders, valuations drawn from a uniform distribution on $[1,2]$, and a signaling function $\Phi(v)=k\sqrt{v}$ with $k>0$. Therefore, the optimal auction uses no reserve price, and participation is fully observable. Following Propositions 4 and 5, the maximal revenue of an auction is then

$$Rev^{A}=\frac{4}{3}+2k\left(\frac{1}{2}\sqrt{\frac{5}{3}}+\frac{1}{2}\sqrt{\frac{4}{3}}\right).$$

Now consider a lottery, in which the object is allocated at random. Provided that both bidders participate, type v's expected utility is $\frac{1}{2}v+k\sqrt{\frac{1}{2}}$. Note that the allocation, i.e., who is assigned the object, does not reveal new information about the bidders' types. To ensure full participation the seller can charge a participation fee of $\frac{1}{2}+k\sqrt{\frac{3}{2}}$, thus revenue is

$$Rev^{P}=1+2k\sqrt{\frac{3}{2}}.$$

Clearly, for sufficiently large k we have $Rev^{P}>Rev^{A}$.[19] Recall that we assume the final allocation (i.e., who gets the object) is observable. Therefore, a mechanism that implements an allocation rule that conditions on reported types necessarily reveals information about bidders. With a concave signaling function, information revealed via the allocation reduces the signaling value the auctioneer can extract. If signaling concerns are sufficiently strong, the gain in signaling value outweighs the loss in terms of standard

[19] Straightforward computations lead to a threshold of $\hat{k}\approx 87.84$. This threshold would reduce with either more bidders, more concave signaling function Φ, or a more spread-out distribution.

revenue $Rev^{\mathcal{M}}$, hence an auction is no longer optimal. Whether a lottery represents an optimal mechanism depends on the marginal gain from improving the allocation versus the marginal loss in terms of signaling value this brings about.[20]

7 Discussion

In this paper, we analyze optimal auctions in an independent private values environment with signaling, i.e. where bidders' care about the perception of a third party. To keep the analysis concise and tractable we focused on linear, convex and concave signaling concerns. The results of Dworczak and Martini (2019) indicate that the disclosure policy maximizing the signaling value for general preferences is a combination of intervals where the type is fully disclosed and intervals on which types are fully pooled. However, it is not straightforward to translate such a disclosure rule into a payment rule for a standard auction. Understanding the polar cases of convexity and concavity allows us to address a preference for the aftermarket that has been studied in the literature on information design, namely where Φ is a distribution function.[21] Under regularity conditions, there is a unique value $\hat{v}$ such that Φ is convex on $[\underline{v}, \hat{v}]$ and concave on $[\hat{v}, \overline{v}]$. Hence, if the participation threshold is sufficiently high we are back in the concave case. Otherwise, maximizing revenue calls for revealing low bids while at the same time pooling higher bids. Disclosure of low value implies that the auctioneer again prefers to disclose whether a bidder participated.

A natural follow-up question concerns the extent to which our results can be generalized to a richer class of mechanisms. Beyond mechanism design, that could provide new and exciting perspectives in applied fields such as advertising, marketing science and industrial organization. For instance, the literature on conspicuous consumption (e.g., Bagwell and Bernheim (1996) and Corneo and Jeanne (1997)) studies product markets where the consumption value depends on the belief of a social contact. A profit-maximizing seller will try to exploit this by tailoring its product line and prices to the information revealed by the consumer's choice. That will lead to new insights about consumer behavior and firm strategies that exploit signaling concerns.[22]

[20] However, note that a formal analysis is all but straightforward. The presence of signaling prevents us from using standard methods, such as pointwise maximization of the objective.

[21] Rayo and Segal (2010) study such a sender–receiver game. The receiver chooses a binary action $a \in \{0, 1\}$. Choosing $a = 0$ yields a fixed utility r which is distributed according to some distribution function G. Choosing $a = 1$ yields utility θ, where θ is the sender's private information. The sender wants to maximize the probability of choosing $a = 1$. Hence, the sender's reduced form utility is $G(\mathbb{E}(\theta))$.

[22] See Rayo and Segal (2010) and Friedrichsen (2018) for analyses into that direction.

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